Geometric Analysis on Euclidean and Homogeneous Spaces

Tufts University

January 8-9 2012

Schedule

		Sunday		Monday		
8.30-9.00		Registration				
9.00-9.50	Gu	inther Uhlmar	ın	Gestur Olafsson		
10.10-11.00	Pe	eter Kuchmen	t	Toshiyuki Kobayashi		
11.00-11.30		Tea		Tea		
11.30-12.20	Т	'oshio Oshima	l	Mike Eastwood		
12.30-1.30	Lunch			Lunch		
1.30-5.40	MA/IP	HA 1	HA 2	MA/IP	НА	
1.30-2.00	Stefanov	Melcher	Ochiai	GHristova	Grinberg	
2.10-2.40	Ambartsoumian	Iosevich	Schlichtkrull	Liu	Pinsky	
2.50-3.20	Steinhauer	Koldobsky	Koldobsky Pasquale Zhou Rubir		Rubin	
3.30-4.00]	Poster social		Poster social		
4.00-4.30						
4.30-5.00	Krishnan	Madych	Gonzalez	Isozaki	Eswarathasan	
5.10-5.40	Vargo	Matsui	Pesenson	Quinto	Nguyen	

Locations

Registration will take place in the foyer of Anderson Hall Morning talks on both Sunday and Monday are in Nelson Auditorium. MA/IP sessions on both Sunday and Monday are in Bromfield-Pearson 101 HA 1 session on Sunday and HA session on Monday are in Bromfield-Pearson 02 HA 2 session on Sunday is in Bromfield-Pearson 07 The poster sessions will be held in the Burden Lounge in Anderson Hall

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Gaik Ambartsoumian (The University of Texas at Arlington)

Exact Inversion of Ultrasound Operators in the Spherical Geometry

In ultrasound tomography an emitter sends ultrasound waves through the body, and the reflections of these waves are registered by a receiver. These data measured for various locations of emitter and receiver are then used to reconstruct the acoustic reflectivity function, which represents an image of the interior of the body. Mathematically this procedure is equivalent to the inversion of an operator, which puts into correspondence to the image function, the measured reflections at available receiver locations. The talk discusses the injectivity of ultrasound operators in the spherical geometry of data acquisition, and exact inversion procedures derived for several setups in this geometry.

Michael Eastwood (Australian National University) The Penrose transform for complex projective space

Complex projective 2-space with its homogeneous Fubini-Study metric has self-dual Weyl curvature whence it enjoys a twistor space and a classical Penrose transform. I shall explain how this goes and how it generalises to higher dimensions. The method is to work with a complexified version known as the double fibration transform, using recent joint work with Joseph Wolf.

Suresh Eswarathasan (University of Rochester)

Eigenfunction supremum bounds for deformations of Schrodinger operators

On a compact Riemannian manifold (M, g), consider the semiclassical Schrödinger operator and corresponding magnetic deformations. We prove an extremal Strichartz estimate for the resulting family of propagators on the L^2 eigenfunctions of the Schrödinger operator.

Fulton Gonzalez (Tufts University)

Multitemporal Wave equations: Mean Value Solutions

Let X = G/K be a Riemannian symmetric space with G semisimple. Let $\Gamma : \mathbb{D}(X) \to \mathbb{D}_W(\mathfrak{a})$ be the Harish-Chandra isomorphism, and let w denote the order of the Weyl group W. For $f_1, \ldots, f_w \in C^{\infty}(X)$, consider the multitemporal system

 $D_x u(x, H) = \Gamma(D)_H u(x, H) \qquad (D \in \mathbb{D}(X))$

for $u \in C^{\infty}(X \times \mathfrak{a})$, with initial data

$$\partial(p_i)_H h(x,0) = f_i(x) \qquad (j = 1, \dots, w).$$

We present mean value solutions to this system in the case when the restricted root multiplicities are even, verifying Huygen's principle.

Eric Grinberg (University of Massachusetts, Boston)

Admissible and inadmissible complexes in integral geometry

The admissibility problem, introduced by I.M.Gelfand, asks for minimal geometries that allow for Radon inversion. The classification of admissible complexes of lines, planes and higher dimensional spaces leads to interesting geometric problems and, in the finite category, some intriguing counting problems. We explore the geometries of admissible and inadmissible complexes and their coupled counting problems in low dimensions with a view towards higher dimensions. The presentation includes joint work with David Feldman and joint work with Mehmet Orhon.

Yulia Hristova (IMA University of Minnesota)

Detection of low emission radiating sources using direction sensitive detectors

In order to prevent influx of highly enriched nuclear material through border checkpoints, new advanced detection schemes need to be developed. Typical issues faced in this context are sources with very low emission against a dominating natural background radiation. Sources are expected to be small and shielded and hence cannot be detected from measurements of radiation levels alone. We consider collimated and Compton-type measurements and propose a detection method that relies on the geometric singularity of small sources to distinguish them from the more uniform background. The method is characterized by high sensitivity and specificity and allows for assigning confidence probabilities of detection. The validity of our approach can be justified using properties of related techniques from medical imaging. Results of numerical simulations are presented for collimated and Compton- type measurements.

Alexander Iosevich (University of Rochester)

Multi-linear generalized Radon transforms and applications to geometric measure theory and related areas

We are going to outline a graph theoretic approach to the development of the theory of multi-linear generalized Radon transforms and apply the resulting estimates to the problems in geometric measure theory inaccessible by the linear methods.

Hiroshi Isozaki (University of Tsukuba)

Inverse scattering on a generalized arithmetic surface

We consider an inverse problem associated with some 2-dimensional non-compact surfaces with conical singularities, cusps and regular ends. Our motivating example is a Riemann surface $\mathcal{M} = \Gamma \setminus \mathbf{H}^2$ associated with a Fuchsian group of the 1st kind Γ containing parabolic elements. \mathcal{M} is then non-compact, and has a finite number of cusps and elliptic singular points, which is regarded as a hyperbolic orbifold. We introduce a class of Riemannian surfaces with conical singularities on its finite part, having cusps and regular ends at infinity, whose metric is asymptotically hyperbolic. By observing solutions of the Helmholtz equation at the cusp, we define a generalized S-matrix. We then show that this generalized S-matrix determines the Riemannian metric and the structure of conical singularties. This is a joint work with Y. Kutylev and M. Lassas.

Toshiyuki Kobayashi (IPMU and University of Tokyo)

Conformally Equivariant Differential Operators and Branching Problems of Verma Modules

The restriction of simple modules of simple Lie algebras with respect to symmetric pairs is nor very simple, and I will discuss some wild feature together with a criterion of "discrete decomposability" as a nice feature.

As its application, I plan to propose a new method to obtain Cohen- Rankin operators for holomorphic automorphic forms and Juhl's conformally equivariant differential operators together with their generalizations.

Alexander Koldobsky (University of Missouri)

Stability in volume comparison problems

We extend the hyperplane inequality in dimensions up to 4 to arbitrary measures in place of volume. To do this, we prove stability in the affirmative part of Zvavitch's solution of the Busemann-Petty problem for arbitrary measures. Then we discuss similar stability results in different settings.

Peter Kuchment (Texas A&M)

Integral geometry and microlocal analysis in the hybrid imaging

The recent appearance of new, so called "hybrid" imaging modalities brings about the prospect for inexpensive and sensitive methods of medical diagnostics. The word "hybrid" reflects the fact that these techniques utilize more than one type of physical waves (e.g., electromagnetic and acoustic). The needs of these new methods lead to various fascinating and hard mathematical problems, which have been attracting attention of many mathematicians for the last decade. Thew issues arising here are often related to, and benefit from the work done on integral geometry and wave equation by Professor S. Helgason. This will be a survey talk.

Venky Krishnan (Tata Institute of Fundamental Research)

A class of singular Fourier integral operators in synthetic aperture radar imaging

We analyze the microlocal properties of the linearized forward scattering operator and the normal operator which arises in synthetic aperture radar imaging for the common midpoint acquisition geometry. When the adjoint operator is applied to the scattered data, artifacts appear. We show that the normal operator can be decomposed as a sum of four operators, each belonging to a class of distributions associated to two cleanly intersecting Lagrangians, thereby explaining the latter artifacts. This is joint work with Gaik Ambartsoumian of University of Texas at Arlington, Raluca Felea of Rochester Institute of Technology, Cliff Nolan of University of Limerick, Ireland and Eric Todd Quinto of Tufts University.

Hongyu Liu (University of California, Irvine) Enhanced Near-cloak by FSH Lining

In this talk, I am going to present our recent study on regularized approximate cloaking for the Helmholtz equation. Various cloaking schemes have been recently proposed and extensively investigated. The existing cloaking schemes in literature are (optimally) within $|\ln \rho|^{-1}$ in 2D and ρ in 3D of the perfect cloaking, where ρ denotes the regularization parameter. In this work, we develop a cloaking scheme with a well-designed lossy layer right outside the cloaked region that can produce significantly enhanced near-cloaking performance. In fact, it is proved that the proposed cloaking scheme could (optimally) achieve ρ^N in \mathbb{R}^N , $N \geq 2$, within the perfect cloaking. It is also shown that the limit of the proposed lossy layer corresponds to a sound-hard layer.

Wolodymyr Madych (University of Connecticut)

Approximate reconstruction from circular and spherical mean Radon transform data

Approximate reconstruction from circular and spherical mean Radon transform data is considered for several detector geometries. We describe two variants of a classical summability type approximate reconstruction method that produce good numerical results and show that in the limit specific cases lead to exact reconstruction. Among the consequences of this development are various inversion type formulas. This work was done in collaboration with Marcus Ansorg, Frank Filbir, and Ruben Seyfried at the Institute of Biomathematics and Biometry, Helmholtz Center Munich, Germany.

Yutaka Matsui (Kinki University)

Topological Radon transforms and their applications

In this talk, we introduce the theory of topological Radon transforms. Note that our topological integration is based on Euler characteristics. We discuss their inversion formulas, some characterizations and applications.

Tai Melcher (University of Virginia)

A quasi-invariance result for heat kernel measures on infinite-dimensional Heisenberg groups

Quasi-invariance of heat kernel measures in infinite dimensions has previously been the subject of much study in elliptic settings. Typically the proofs in the elliptic case rely on lower bounds on the Ricci curvature. Of course, such lower bounds are unavailable in a subelliptic setting. In this talk, we will discuss Cameron-Martin type quasi-invariance results for subelliptic heat kernel measures on infinite-dimensional Heisenberg-like groups. The main ingredient in the proof is a generalized curvature-dimension estimate which holds on approximating finite-dimensional projection groups.

Linh Nguyen (University of Idaho)

Range description for a spherical mean transform on spaces of constant curvatures

In this talk, we describe the range of a restricted spherical mean transform, which sends a function supported inside a closed geodesics ball in a hyperbolic space to its mean values on the geodesics spheres centered at the boundary of the ball. The description resembles that of the same transform on the Euclidean spaces, which was obtained by M. Agranovsky, D. Finch, and P. Kuchment [Inverse Problems and Imaging, 3(3):373–382, 2009] and M. Agranovsky and L. Nguyen [J. Anal. Math., 112:351–367, 2010].

We also present a similar characterization for the corresponding transform on the two dimensional spherical space.

Hiroyuki Ochiai (Kyushu University)

Positivity of an alpha determinant

The alpha determinant is one of one-parameter interpolations of the determinant and the permanent. This interpolation is different from the quantum determinant, and has some role in statistics and probability, and recently in representation theory of general linear groups. In this talk, we will discuss the problem raised by T. Shirai and Y. Takahashi on the positivity of the alpha determinant on the positive cone of positive definite hermitian matrices. The alpha determinant of some special matrix has an expression in terms of the special values of the generalized hypergeometric series ${}_{3}F_{2}$ by 3F2 and this fact enables us to determine the signature of the alpha determinant by an estimate of such an special value. The proof relies on a property of the Radon transform.

Gestur Ólafsson (Louisiana State University)

Spherical functions on limits of compact symmetric spaces.

Spherical representations and functions are the building blocks for harmonic analysis on Riemannian symmetric spaces. We will give a short overview of injective limits of compact symmetric spaces $G_{\infty}/K_{\infty} = \varinjlim G_n/K_n$ and limits of spherical representations. We will then describe what happens to the limits of the related spherical $\varphi_n(x) = \langle e_n, \pi_n(x)e_n \rangle$ where e_n is a K_n -fixed unit vector for π_n . We will show that the limit $\lim_{t\to\infty} \varphi_n(x)$ defines a spherical function on G_{∞}/K_{∞} if and only if the rank of G_n/K_n is bounded. This is a joint work with M. Dawson and J. Wolf.

Toshio Oshima (University of Tokyo)

Generalizations of Radon transforms on compact homogeneous spaces

After the work by Funk studying the integration of functions on a sphere along the great circles, several generalizations have been established. In this talk we discuss integrations of functions on generalized flag manifolds along nice submanifolds. Moreover we consider "generalized flag manifolds for a finite set". Some examples and open problems will be given.

Angela Pasquale (University of Metz)

Uncertainty principles for the Schrödinger equation on Riemannian symmetric spaces of the noncompact type.

Let X be a Riemannian symmetric space of the noncompact type. We prove that the solution of the time-dependent Schrödinger equation on X with square integrable initial condition f is identically zero at all times t whenever f and the solution at a time $t_0 > 0$ are simultaneously very rapidly decreasing. The stated condition of rapid decrease is of Beurling type. Conditions respectively of Gelfand-Shilov, Cowling-Price and Hardy type are deduced. This is a joint work with Maddala Sundari (Chennai Mathematical Institute).

Isaac Z. Pesenson (Temple University)

Band-limited Localized tight frames on Compact Homogeneous Manifolds

The goal of the talk is to explain construction of band-limited and highly localized tight frames on general compact homogeneous manifolds. The construction can be considered as an analogue of the well-known φ -transform on Euclidean spaces. This presentation is based on a joint paper with the close friend and excellent mathematician Daryl Geller (12/26/1950-01/27/2011).

Mark A. Pinsky (Northwestern University)

Can you feel the shape of a manifold with Brownian motion

We review the formulation of Brownian motion as the weak limit of piecewise geodesic paths. The mean exit time and exit place from a geodesic ball can be studied in terms of asymptotic expansions in the radius of the ball. In dimensions less than six, the mean exit time of Euclidean space characterizes Euclidean space. In dimensions greater than six, counter-examples show that distinct riemannian metrics may give rise to the same exit time distribution. Parallel results are obtained for principal Dirichlet eigenvalue of a small ball.

Todd Quinto (Tufts University)

Algorithms in bistatic ultrasound

The speaker will describe a problem in bistatic ultrasound that reduces to integrals over ellipses with foci a fixed distance apart. In this problem, the foci rotate around a circle and the goal is to reconstruct the structure of an object inside the circle. The speaker will describe theorems of the authors that explain how the reconstruction operator images singularities and he will show reconstructions from this problem.

This is joint work with Gaik Ambartsoumian, Venkateswaran Krishnan, and Howard Levinson

Boris Rubin (Louisiana State University)

Inversion Formulas for Spherical Means in Constant Curvature Spaces

The following problem arises in thermoacoustic tomography and has intimate connection with PDEs and integral geometry: Reconstruct a function f supported in an n-dimensional ball B, if the spherical means of f are known over all geodesic spheres centered on the boundary of B. We propose a new approach to this problem, which yields explicit reconstruction formulas in arbitrary constant curvature space, including the Euclidean space \mathbb{R}^n , the *n*dimensional sphere, and the hyperbolic space. The main idea is analytic continuation of the corresponding operator families. The results can be applied to inverse problems for Euler-Poisson-Darboux equations in constant curvature spaces of arbitrary dimension. This is a joint work with Yuri A. Antipov and Ricardo Estrada.

Henrik Schlichtkrull (University of Copenhagen)

A uniform bound on the matrix elements of the irreducible representations of SU(2)

With natural choices of basis vectors all matrix elements of the irreducible unitary representations of SU(2) are explicitly expressible in terms of Jacobi polynomials. Let $g_n^{a,b}(x)$ be such a matrix element (with non-negative integers a, b, n) on a torus, parametrized by $x = \cos\theta$. Then

$$|g_n^{a,b}(x)| \le Cd^{-1/4}(1-x^2)^{-1/4}$$

where d = 2n + a + b + 1 is the dimension of the corresponding irreducible representation, and where C is uniform in all the parameters. Previously, the corresponding bound for Jacobi polynomials was known only under extra conditions on a,b,n or x.

This is joint with U. Haagerup.

Plamen Stefanov (Purdue University)

The Identification problem in SPECT: uniqueness, non-uniqueness and stability

We study the problem of recovery both the attenuation a and the source f in the attenuated X-ray transform in the plane. We study the linearization as well. It turns out that there is a natural Hamiltonian flow that determines which singularities we can recover. If the perturbation δa is supported in a compact set that is non-trapping for that flow, then the problem is well posed. Otherwise, it may not be, and at least in the case of radial a, f, it is not. We present uniqueness and non-uniqueness results both for the linearized and the non-linear problem; as well as a Hölder stability estimate.

Dustin Steinhauer (Texas A&M)

Inverse Problems in Medical Imaging with Internal Information

Several hybrid imaging methods (e.g. those combining electrical impedance or optical imaging with acoustics) enable one to obtain internal information. This information stabilizes the exponentially unstable modalities of optical and electrical impedance tomography. A number of manifestations of this effect have been studied previously. I will discuss a simple, general approach that shows when and why internal information stabilizes the reconstruction. (This talk presents joint work with Peter Kuchment.)

Gunther Uhlmann (University of Washington)

Travel Time Tomography and Tensor Tomography

We will give a survey on some recent results on travel tomography which consists in determining the index of refraction of a medium by measuring the travel times of sound waves going through the medium. We will also consider the related problem of tensor tomography which consists in determining a function, a vector field or tensors of higher rank from their integrals along geodesics.

Jim Vargo (Texas A&M) The Spherical Mean Problem

Given a function, consider its integrals over the set of all spheres with centers on a hypersurface S. The spherical mean problem is to invert this transform by reconstructing the function from such integrals. It remains an open problem to characterize the hypersurfaces S for which the transform is injective. We will discuss new research on this problem.

Ting Zhou (University of Washington)

On approximate cloaking by nonsingular transformation media

In a joint work with H. Y. Liu, we gave a comprehensive study on regularized approximate electromagnetic cloaking in the spherical geometry via the transformation optics approach. The following aspects were investigated: (i) near-invisibility cloaking of passive media as well as active/radiating sources; (ii) the existence of cloak-busting inclusions without lossy medium lining; (iii) overcoming the cloaking-busts by employing a lossy layer outside the cloaked region; (iv) the frequency dependence of the cloaking performances. We address these issues and connect the obtained asymptotic results to singular ideal cloaking. Numerical verifications and demonstrations are provided to show the sharpness of our analytical study. In another work with M. Lassas, we considered truncation based regularized 2D acoustic cloaking. We showed that, surprisingly, a non-local boundary condition appears on the inner cloak interface. This effect in the two dimensional (or cylindrical) invisibility cloaks, which seems to be caused by the infinite phase velocity near the interface between the cloaked and uncloaked regions, is very different to the earlier studied behavior of the solutions in the three dimensional cloaks.

Matthew Dawson (Louisiana State University)

Direct Systems of Spherical Representations and Compact Riemannian Symmetric Spaces

Spherical analysis plays a critically important role in the harmonic analysis of Gelfand Pairs. It is natural to try to extend this machinery to infinite-dimensional versions of these spaces. Olshanski and Faraut have considered spherical functions on direct limits (G_{∞}, K_{∞}) of Gelfand Pairs $(G_k, K_k), k \in \mathbb{N}$. It has been unclear whether one can obtain a K_{∞} -spherical representation of G_{∞} by taking limits of spherical representations of the G_k . We show that for the case when (G_k, K_k) is a direct system of compact Riemannian symmetric spaces, the direct limit of spherical representations of the groups G_k will be a spherical representation of G if and only if the rank of (G_{∞}, K_{∞}) is finite (equivalently, it is the Grassmann manifold of p-planes in \mathbb{F}^{∞} where $p < \infty$ and \mathbb{F} is \mathbb{R} , \mathbb{C} or \mathbb{H}). This is joint work with G. Olafsson and J. Wolf.

Daniel Fresen (University of Missouri)

Concentration inequalities for random polytopes

We study the convex hull of a large i.i.d. sample from a general log-concave distribution with non-vanishing density. We show that with high probability, this convex hull (called a random polytope) approximates a pre-determined convex body called the floating body. This is a multivariate version of the Gnedenko law of large numbers which applies to the one dimensional case of the maximum of a large i.i.d. sample.

Vivian Ho (Louisiana State University)

Paley-Wiener Theorem for Line Bundles over Compact Symmetric Spaces

We generalize a Paley-Wiener theorem to homogeneous line bundles L_{χ} on a compact symmetric space U/K with χ a character of K. The Fourier coefficients of a spherical function f of type χ are defined by integration of f against the elementary χ -spherical functions, depending on a spectral parameter μ , which in turn parametrizes the χ -spherical representations π of U. The Paley-Wiener theorem characterizes f with sufficiently small support in terms of holomorphic extendability and exponential growth of their Fourier transform.

Koichi Kaizuka (University of Tsukuba)

Uniform resolvent estimates on symmetric spaces of noncompact type

We discuss uniform resolvent estimates for elliptic Fourier multipliers and for the Schroedinger operator with real-valued small potentials on symmetric spaces of noncompact type. Then we obtain time-global smoothing estimates for corresponding dispersive equations. For low-frequency estimates, a pseudo-dimension appears as an upper bound of the order of elliptic Fourier multipliers.

Toshihisa Kubo (Oklahoma State University)

Systems of second-order invariant differential operators of non-Heisenberg parabolic type

The wave operator \Box in Minkowski space $\mathbf{R}^{3,1}$ is a classical example of a conformally invariant differential operator. The Lie algebra $\mathfrak{so}(4,2)$ acts on $\mathbf{R}^{3,1}$ via a multiplier representation σ . When acting on sections of an appropriate bundle over $\mathbf{R}^{3,1}$, the elements of $\mathfrak{so}(4,2)$ are symmetries of the wave operator \Box ; that is, for $X \in \mathfrak{so}(4,2)$, we have

$$[\sigma(X),\Box] = C(X)\Box$$

with C(X) a smooth function on $\mathbf{R}^{3,1}$.

The notion of conformal invariance for a differential operator appears implicitly and explicitly in the literature. The conformality of one operator has been generalized by Barchini-Kable-Zierau to systems of differential operators. Such systems yield homomorphisms between generalized Verma modules. In this presentation we build such systems of second-order differential operators in the maximal non-Heisenberg parabolic setting. We also discuss the corresponding homomorphisms between generalized Verma modules.

Kyung-Taek Lim (Tufts University)

Surjectivity and range description of the single radius spherical mean on Euclidean space

We show the surjectivity of the single radius spherical mean from $\mathcal{E}(\mathbb{R}^n)$ to itself and from $\mathcal{D}'(\mathbb{R}^n)$ to itself. We also present range descriptions of the signle radius spherical mean on $\mathcal{E}'(\mathbb{R}^n)$ and on $\mathcal{D}(\mathbb{R}^n)$. For these tasks, we apply Hörmander's work on convolution equations.

Carlos Montalto (Purdue University)

Stable determination of generic simple metrics, vector field and potencial from the hyperbolic Dirichlet-to-Neumann map

Lets consider a compact Riemmanian manifold M with non-empty boundary ∂M . We consider the inverse problem for the second order hyperbolic initial-boundary value problem

$$\partial_t^2 u + P(x, D)u = 0 \quad [0, T] \times M,$$

$$u|_{t=0} = \partial_t u|_{t=0} = 0 \quad x \in M,$$

$$u|_{(0,T) \times \partial M} = f,$$

where P(x, D) is a first-order perturbation of the Laplace operator $-\Delta_g$ on (M, g). More explicitly, P(x, D) is a second order uniformly elliptic self-adjoint operator with real principal part; any such operator can be written in local coordinates as

$$P(x,D) = -\frac{1}{\sqrt{\det g}} \left(-\frac{\partial}{\partial x^j} + \mathrm{i} \, b_j \right) g^{ij} \sqrt{\det g} \left(-\frac{\partial}{\partial x^i} + \mathrm{i} \, b_i \right) + q,$$

for b real-valued co-vector field and q real-valued function on M.

We prove Hölder type stability estimates near generic simple Riemannian metrics for the inverse problem of recovering such metrics g, the covector field b, and the potencial q from the hyperbolic Dirichlet-to-Neumann map associated, $f \rightarrow \partial_{\nu} u^{f} - i \langle \nu, b \rangle_{g} u|_{\partial M \times [0,T]}$. Here $\partial_{\nu} u^{f}$ is the normal derivative of the solution of the initial-value problem.

Vignon Oussa (Saint Louis University)

Bandlimited Spaces on Some 2-step Nilpotent Lie Groups With One Parseval Frame Generator

Let N be a step two connected and simply connected non commutative nilpotent Lie group which is square-integrable modulo the center. Let Z be the center of N. Assume that $N = P \rtimes M$ such that P and M are simply connected, connected abelian Lie groups, M acts non trivially on P by automorphisms and dim $P/Z = \dim M$. We study band-limited subspaces of $L^2(N)$ which admit Parseval frames generated by discrete translates of a single function. We prove that it is always possible to find some quasi-lattice and a function generating a Parseval frame. We also give explicit algorithms for the construction. Furthermore, we find characteristics of band-limited subspaces of $L^2(N)$ which do not admit a single Parseval frame. We also provide some conditions under which a continuous wavelet transform related to the left regular representation admits discretization by some discrete set $\Gamma \subset N$. Finally, we show some explicit examples.

Patrick Spencer (University of Missouri)

Lorentz Balls Are Not Intersection Bodies

The unit ball of the n-dimensional Lorentz space $l_{w,q}^n$ is not an intersection body for q>2 and $n\geq 5$.