An optimization framework for one-step spectral CT image reconstruction and current challenges: Three material decomposition

Emil Sidky[1], Rina Foygel Barber[2], Taly Gilat-Schmidt[3], and Xiaochuan Pan[1]

[1]University of Chicago - Dept. of Radiology

[2]University of Chicago - Dept. of Statistics

[3]Marquette University - Dept. of Biomedical Engineering

Work supported by
NIH: NCI and NIBIB
NSF

Allan Cormack on the Radon Transform

1973 Physics in Medicine and Biology

"It has recently come to the author’s attention that the problem of determining a function in a plane from its line integrals was first solved by J. Radon in 1917 (and not in the nineteenth century, as suggested in Cormack (1963))"

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1963 Journal of Applied Physics

"Is it possible to determine g? One would think that this problem would be a standard part of the nineteenth century mathematical repertoire, but the author has found no reference to it in standard works."

Outline

One-step spectral CT image reconstruction (OSSCIR)

MOCCA algorithm & block-diagonal step-preconditioning

Spectral calibration

Studies with multi-material basis decomposition

- Experimental results for objects with metal and Gadolinium contrast agent

Spectral CT model
materials decomposition

$$I_{w,s,u} = \int dE\, S_{w,u}(E) \exp\left[- \int_{\ell(s,u)} dt\, \mu(E, \vec{r}(t)) \right]$$

$$\mu(E, \vec{r}(t)) = \sum_m \left(\frac{\mu_m(E)}{\rho_m} \right) \rho_m f_m(\vec{r}(t))$$

photon-counting model: $$\hat{c}_{w,l}(f_{km}) = S_{w,u,i} \exp\left[-\mu_{im} X_{lk} f_{km} \right]$$

w - energy window
l - transmission ray, depends on *s,u*
i - energy index
m - material label
k - pixel index

Spectral CT model with spectral scaling

photon-counting model: $\hat{c}_{w,l}(f) = S_{w,u,i} \exp\left[-\mu_{im} X_{lk} f_{km}\right]$

photon-counting model with scaling:

$$\hat{c}_{w,l}(f, \alpha) = S_{w,u,i} \exp\left[-\alpha_{w,u} - \mu_{im} X_{lk} f_{km}\right]$$

Constrained one-step spectral CT image reconstruction (cOSSCIR)

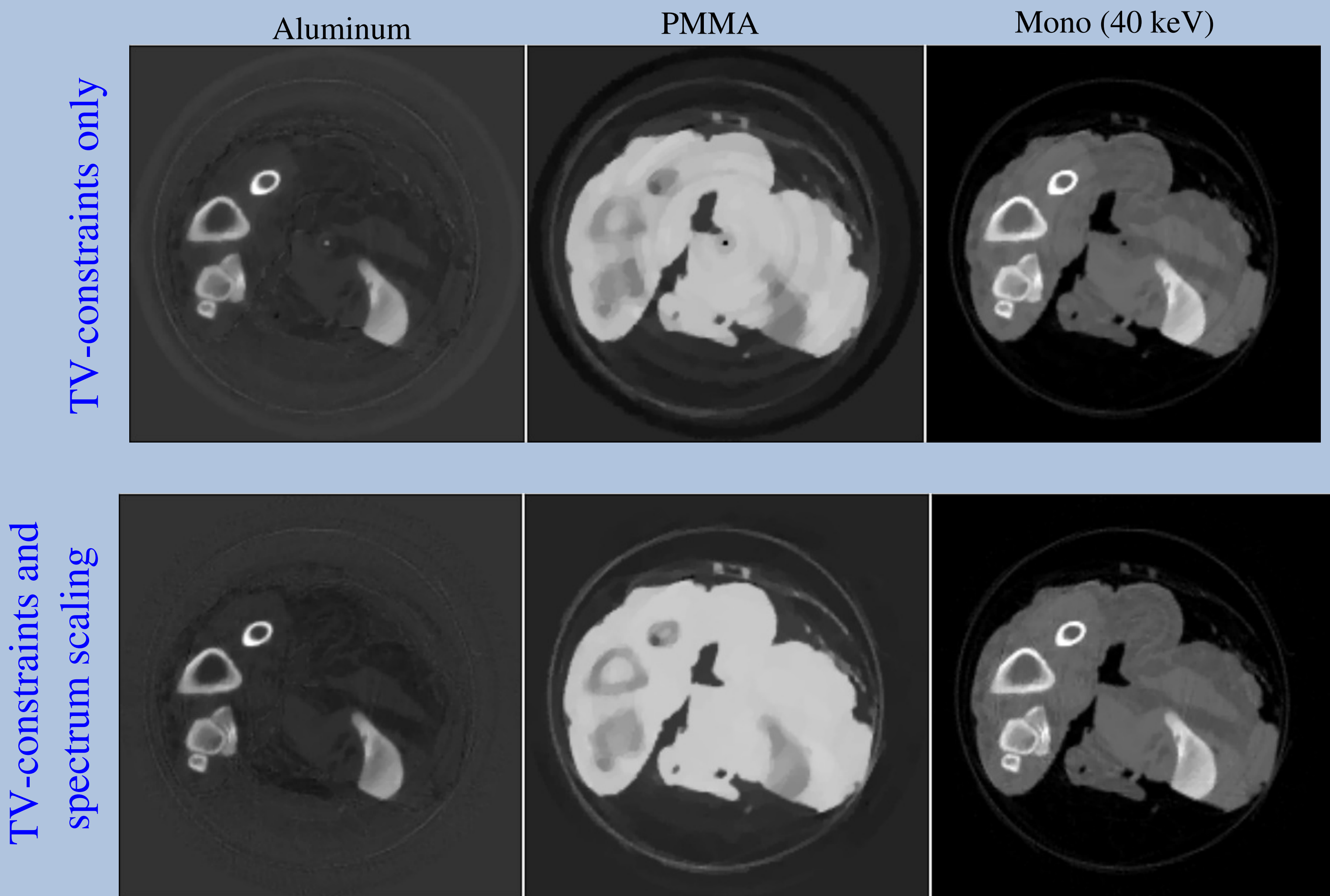
$$f^\star, \alpha^\star = \mathrm{argmin} D_{\mathrm{TPL}}(c, \hat{c}(f, \alpha))$$

$$\text{constraints} \;\; \|\alpha\|_2 \leq \delta; \;\; \|\nabla f'_m\|_1 \leq \gamma_m$$

* D_{TPL} is a nonconvex function
  - use mirrored convex/concave (MOCCA) alg.

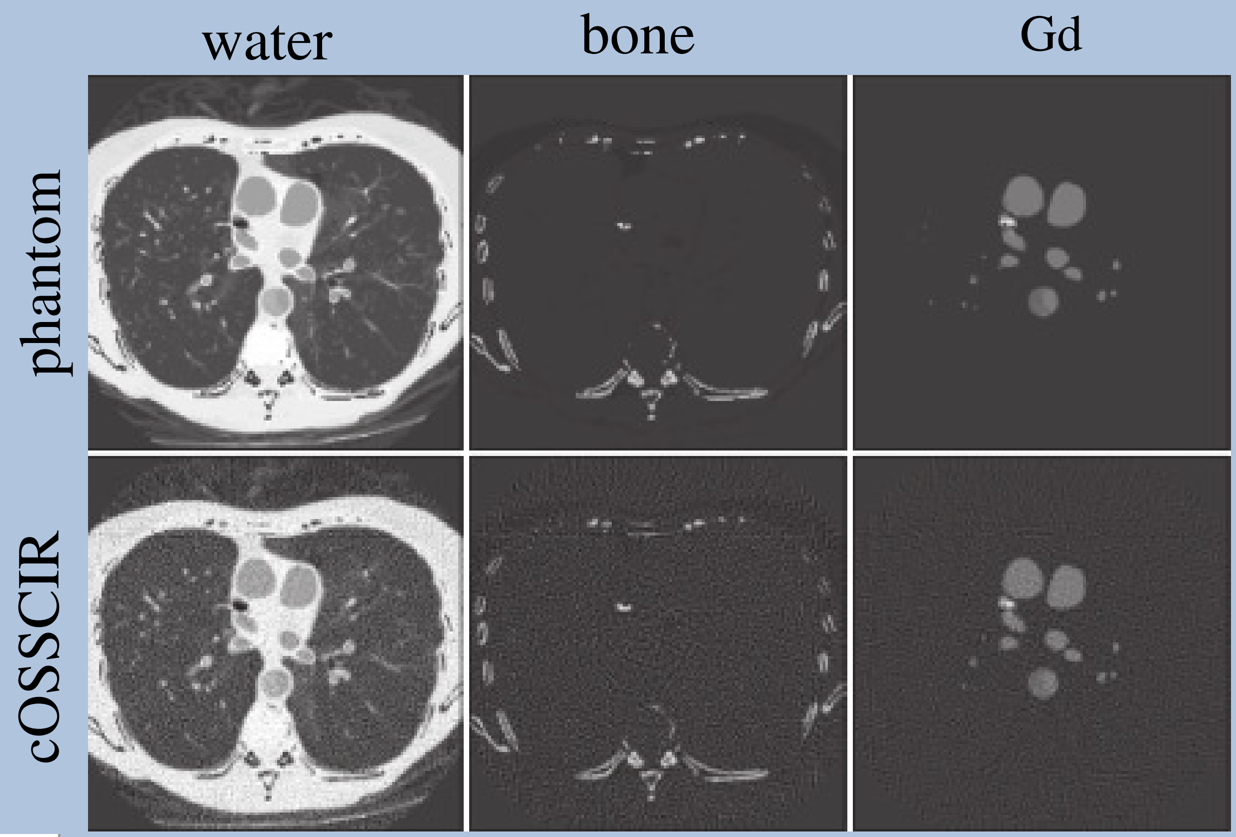
* constraint parameters
  - use validation

Ring artifact mitigation with partial spectrum auto-calibration

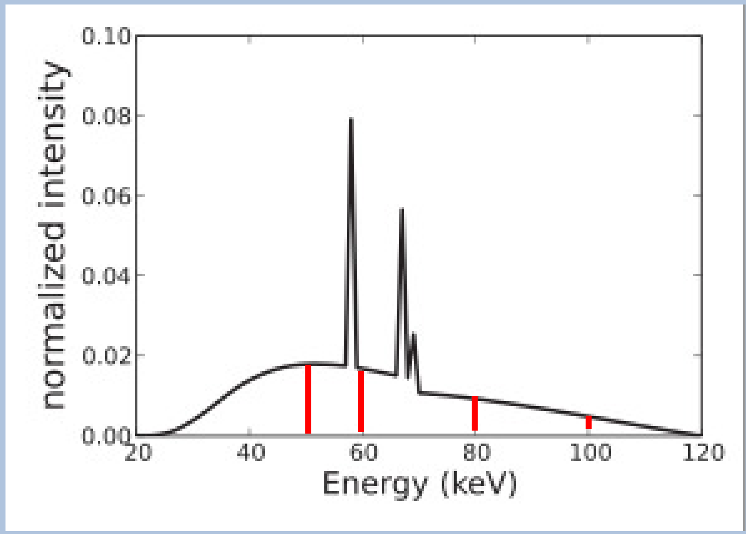


Schmidt et al., TMI vol. 36, 1808-1819 (2017)

cOSSCIR and contrast agent imaging

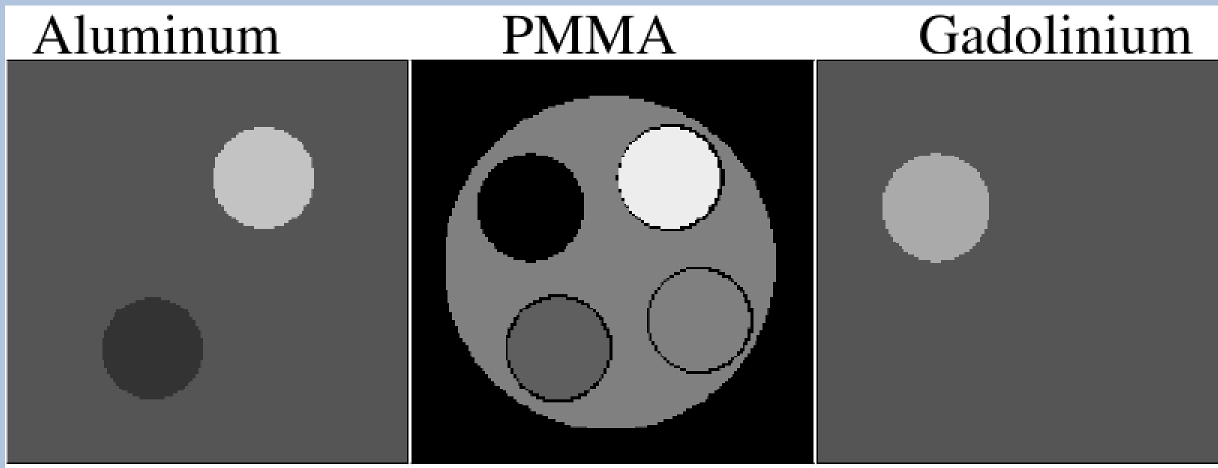


Ideal photon-counting spectral response

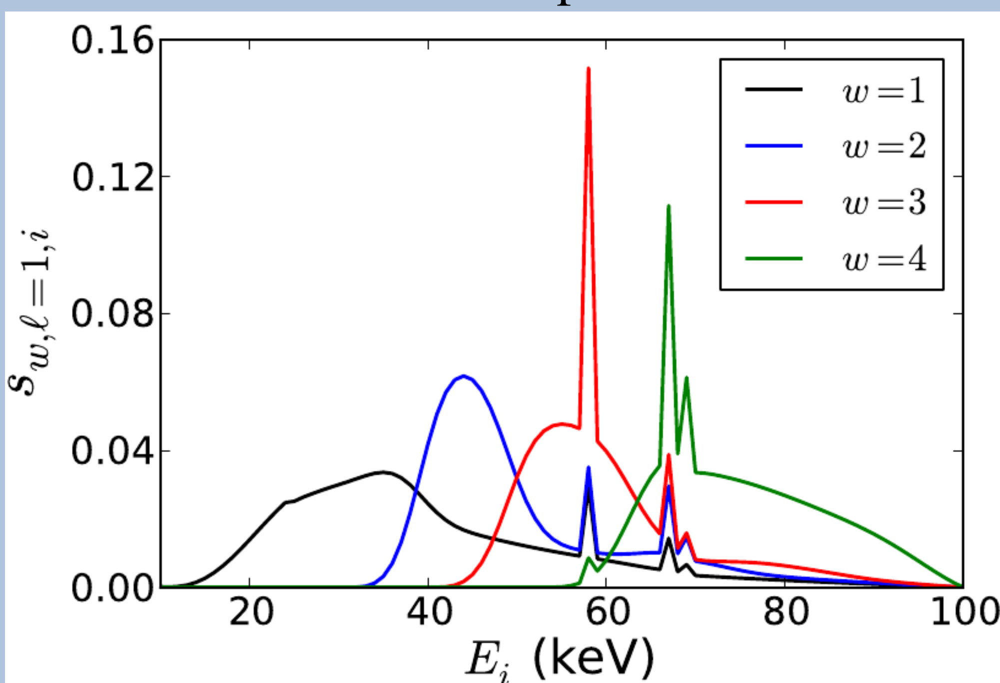


Barber et al., PMB vol. 61, 3784-3818 (2016)

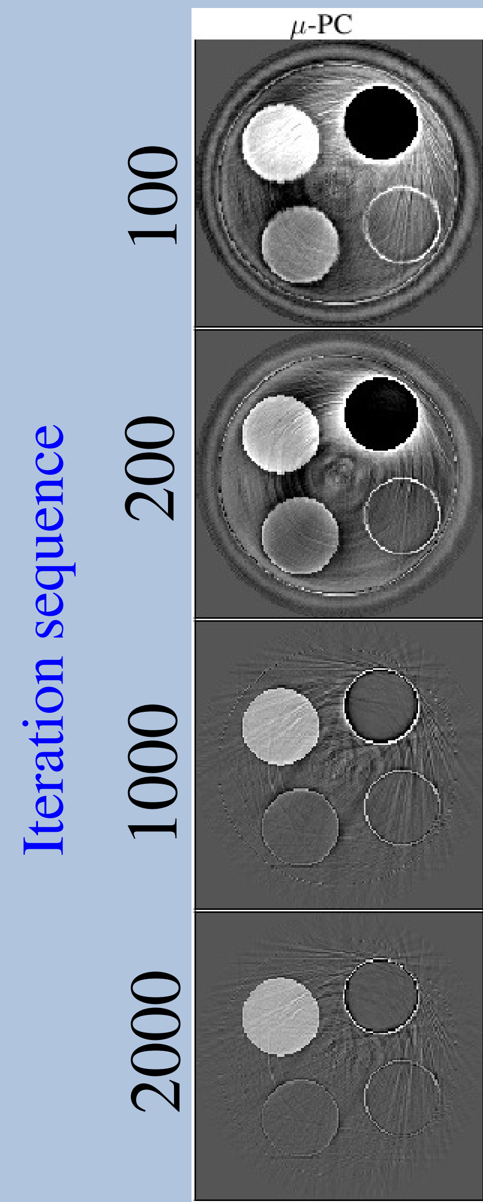
Rods phantom with realistic spectra



realistic spectra



D_{TPL} minimization
noiseless data



MOCCA algorithm & block-diagonal step-preconditioning

Chambolle-Pock primal-dual algorithm summary

optimization problems:

$$x^\star = \text{argmin } \{F(Kx) + G(x)\}$$

$$\boxed{x^\star = \text{argmin } F(Kx)}$$

$$\text{min}_x \text{max}_y \left\{ y^T K x - F^*(y) \right\}$$

pseudocode:

$$y_{n+1} = \text{prox}_\Sigma[F^*](y_n + \Sigma K \overline{x}_n)$$

$$x_{n+1} = x_n - T K^T y_{n+1}$$

$$\overline{x}_{n+1} = x_{n+1} + \theta(x_{n+1} - x_n)$$

step-size matrices: $\begin{pmatrix} T^{-1} & -K^\top \\ -K & \Sigma^{-1} \end{pmatrix}$ is PSD

<u>two step-size choices</u>

$$\sigma = (1/\lambda)/\|K\|_2 \quad \tau = \lambda/\|K\|_2$$

$$\Sigma = (1/\lambda)\ \text{diag}(|K|\mathbf{1})^{-1} \quad T = \lambda\ \text{diag}(|K^\top|\mathbf{1})^{-1}$$

definitions: $F^* = \max \left\{ x'^T x - F(x') \right\}$

$$\text{prox}_\Sigma[F^*](x) = \text{argmin } \left\{ F^*(x') + \frac{1}{2}(x - x')^\top \Sigma^{-1}(x - x') \right\}$$

MOCCA

optimization problem: $x^\star = \text{argmin } F(Kx)$

F(Kx) is a convex function of *x*

F(y) is convex-concave:

$F(y) = F_+(y) - F_\_(y)$,

where $F_+(y)$ and $F_\_(y)$ are convex

$$\boxed{F(Kx) \neq \max_y \left\{ y^T Kx - F^*(y) \right\}}$$

MOCCA step sizes

$$L_{\mathrm{LSQ}}(f) = \frac{1}{2}\|Xf - g\|_2^2$$

$$\partial L_{\mathrm{LSQ}}(f) = X^\top (Xf - g) = X^\top r_{\mathrm{LSQ}}(f)$$

$$L_{\mathrm{TPL}}(f) = D_{\mathrm{TPL}}(c, \hat{c}(f))$$

$$\partial L_{\mathrm{TPL}}(f) = X^\top \mu^\top A(f)^\top r_{\mathrm{TPL}}(f)$$

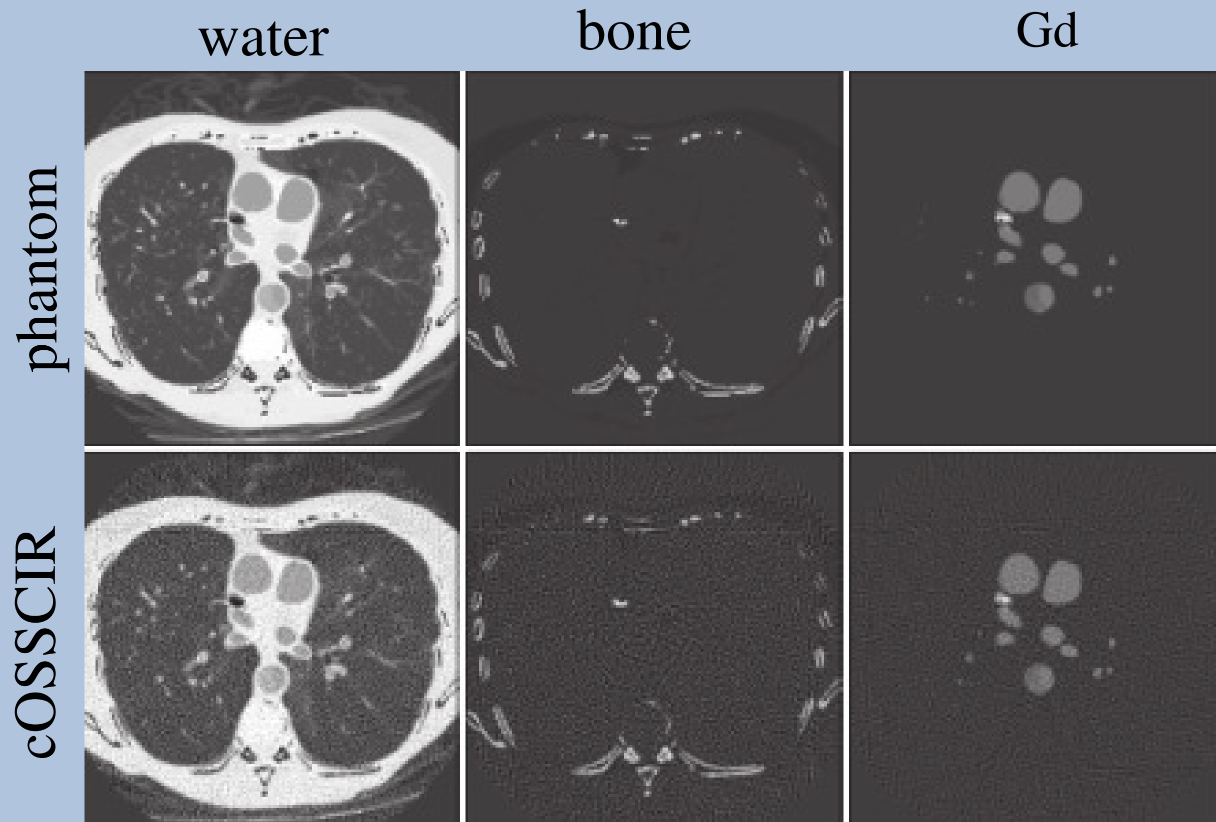
$$K(f) = A(f)\mu X$$

$$\Sigma = (1/\lambda)\ \mathrm{diag}(|K(f)|\mathbf{1})^{-1} \quad T = \lambda\ \mathrm{diag}(|K^\top(f)|\mathbf{1})^{-1}$$

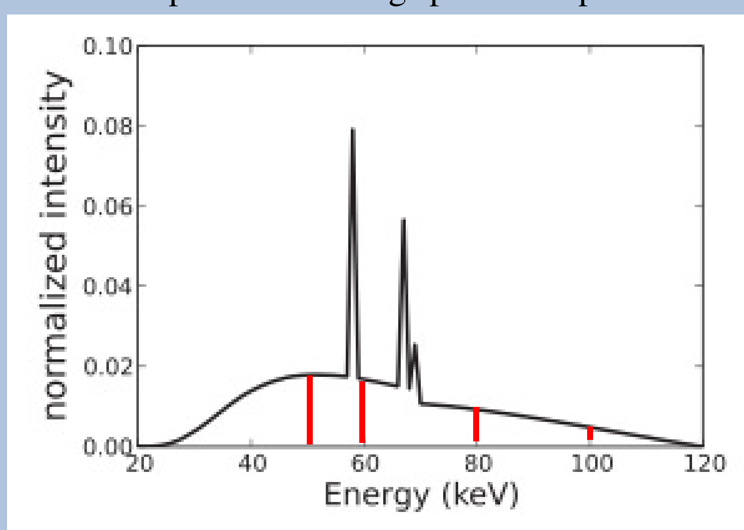
$$K_{w\ell,mk}(f)$$

w - energy window
l - transmission ray
m - material label
k - pixel index

Chest phantom with ideal spectra

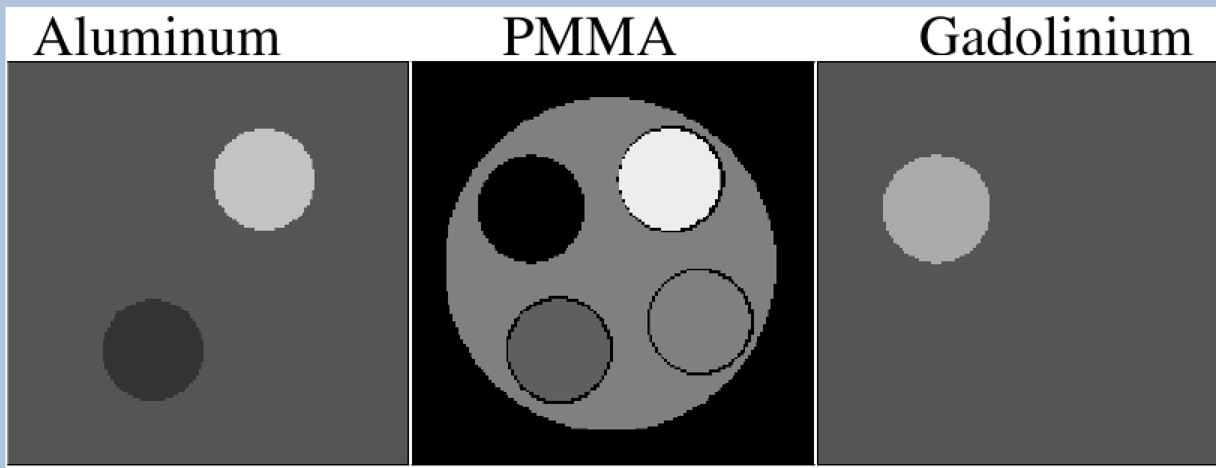


Ideal photon-counting spectral response

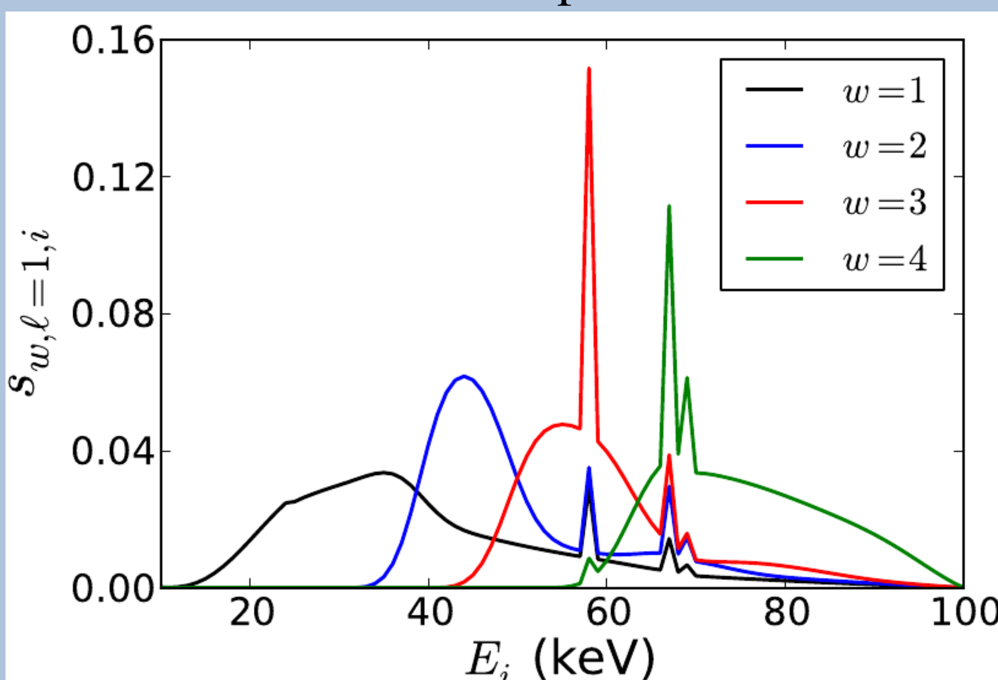


results obtained within 1,000 iterations
using μ-preconditioning

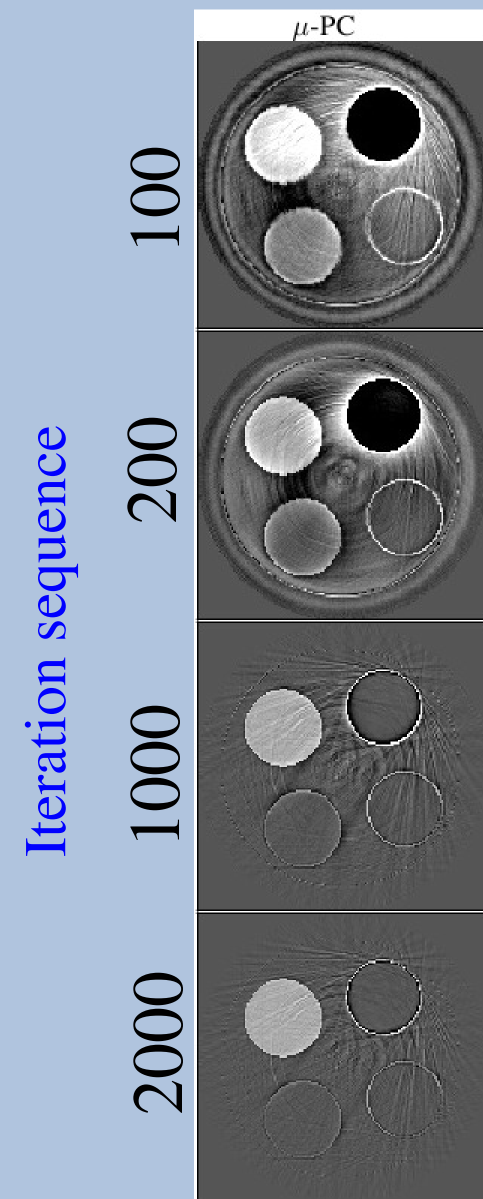
Rods phantom with realistic spectra



realistic spectra



D_{TPL} minimization
noiseless data



Block-diagonal step-preconditioning

CP step-preconditioner

$$K(f) = A(f)\mu X$$

$$\Sigma = (1/\lambda)\ \mathrm{diag}(|K(f)|\mathbf{1})^{-1} \quad T = \lambda\ \mathrm{diag}(|K^\top(f)|\mathbf{1})^{-1}$$

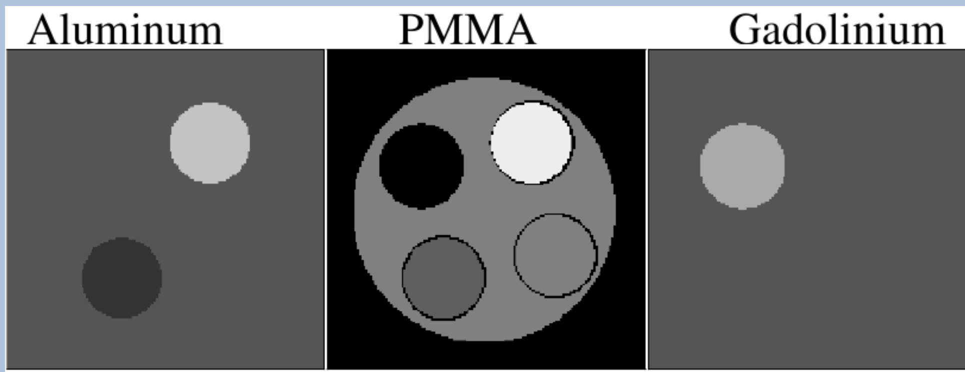
proposed CP step-preconditioner

$$\left(\Sigma^{-1}\right)_{w\ell,w'\ell'} = \lambda \sum_k \sqrt{\sum_m K^2_{w\ell,mk}(f_0)}\ \mathbf{I}_{w\ell,w'\ell'}$$

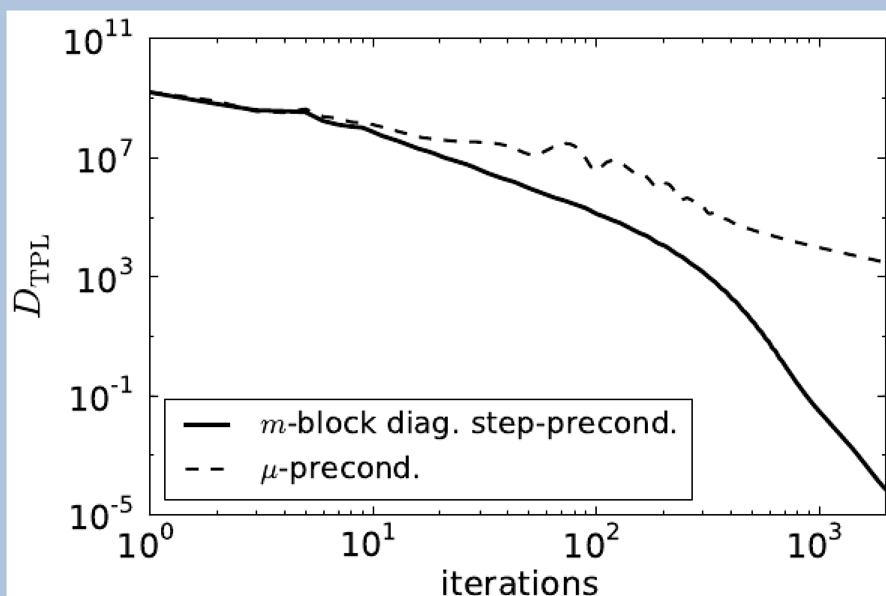
$$\left(T^{-1}\right)_{mk,m'k'} = \frac{1}{\lambda} \sum_{w,\ell} \frac{K_{w\ell,mk}(f_0) K_{w\ell,m'k}(f_0)}{\sqrt{\sum_{m''} K^2_{w\ell,m''k}(f_0)}} \mathbf{I}_{k,k'}$$

invariance to orthogonal transformation

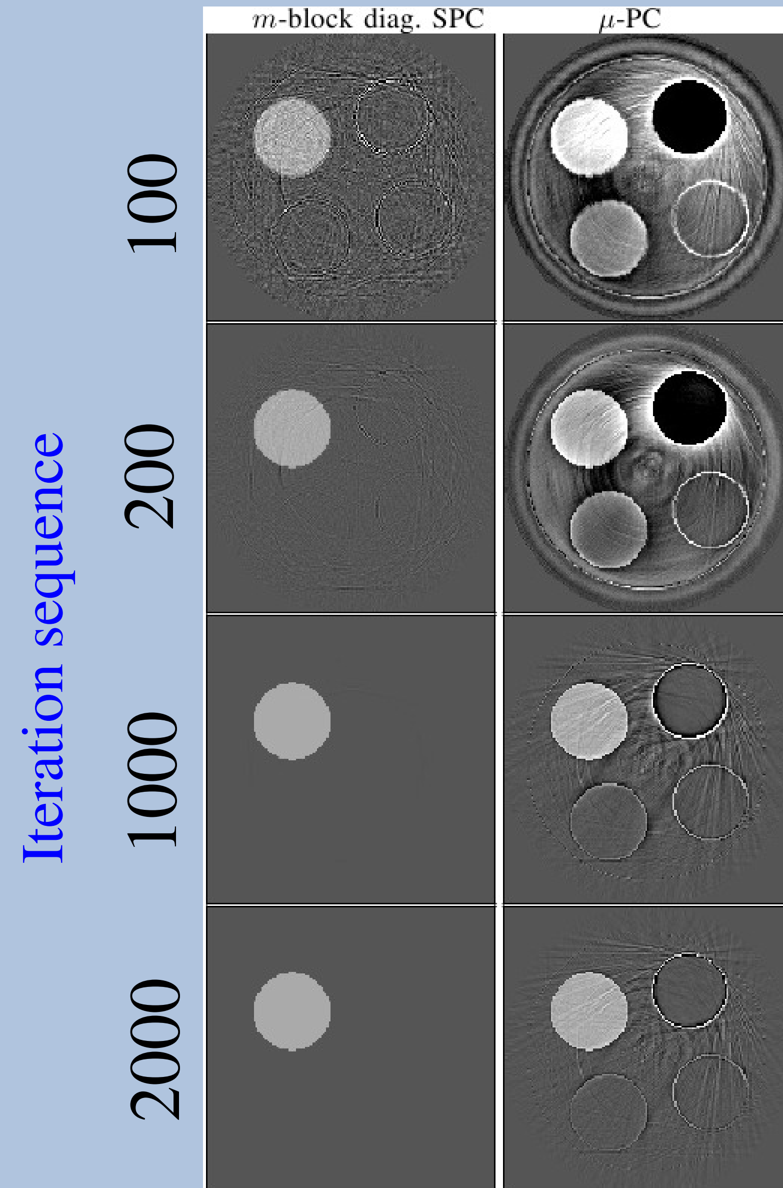
Comparison of μ-PC and BD-SPC



Convergence comparison



D_{TPL} minimization
noiseless data

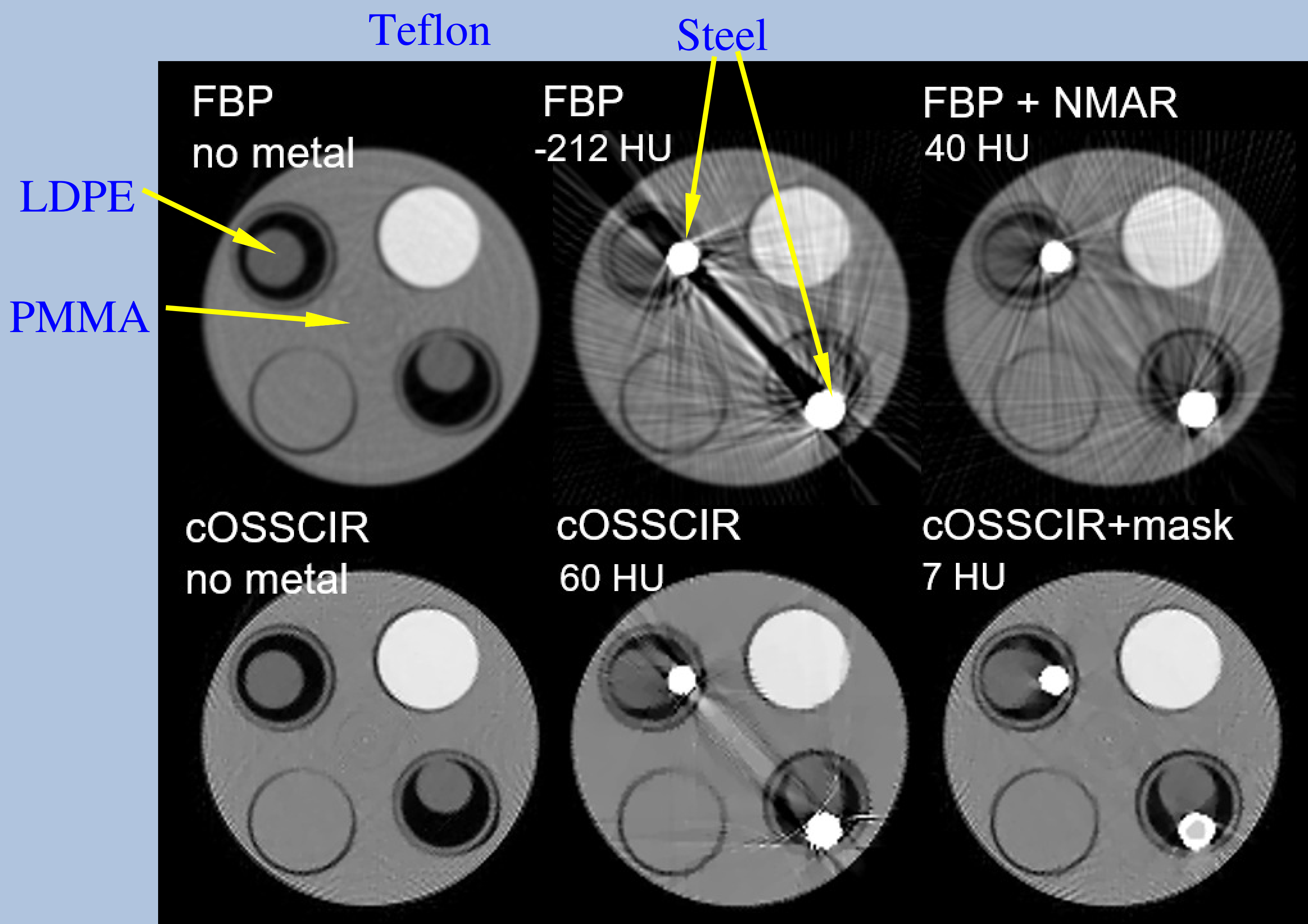


Multi-basis-material projects

* Contrast agent imaging
    - experiment with Gd imaging (3 materials)
    - simulation with Gd and I imaging (4 materials)

* Spectral CT with metal
    - experiment with decomposition
      into water, bone, and stainless steel

Rod phantom experiment with metal



4-window transmission data. Masked: measurements with <10 counts eliminated.

Other algorithm details

photon-counting model: $\hat{c}_{w,l}(f_{km}) = S_{w,u,i}\ \text{softexp}\left[-\mu_{im} X_{lk} f_{km}\right]$

$$\text{softexp}(x) = \begin{cases} \exp(x) & x \leq 0 \\ x+1 & x > 0 \end{cases}$$

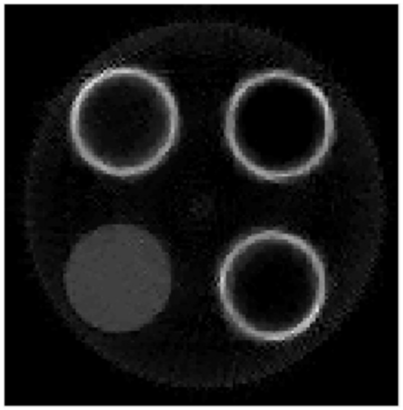
Group total variation (GTV):

$$f^{\star}, \alpha^{\star} = \text{argmin} D_{\text{TPL}}(c, \hat{c}(f, \alpha))$$

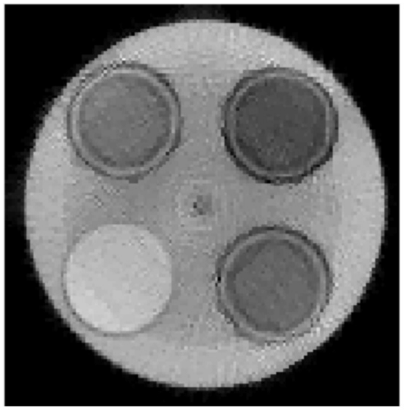
$$\text{constraints}\ \ \|\alpha\|_2 \leq \delta;\ \sum_m \|\nabla f'_m\|_1 \leq \gamma$$

System calibration

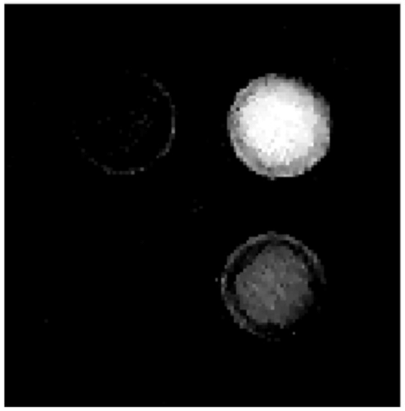
Images for GTV=2000



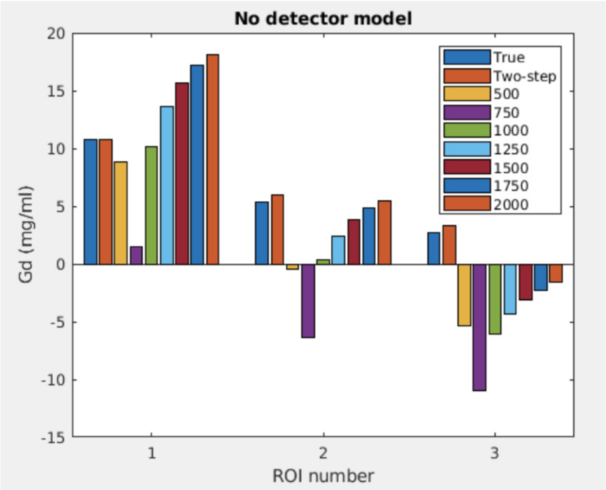
Aluminum [0, 0.5]

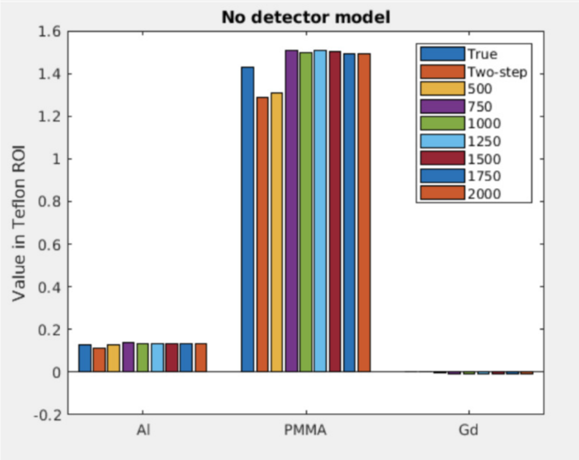


PMMA [0, 2]



Gd [0, .02]





System calibration

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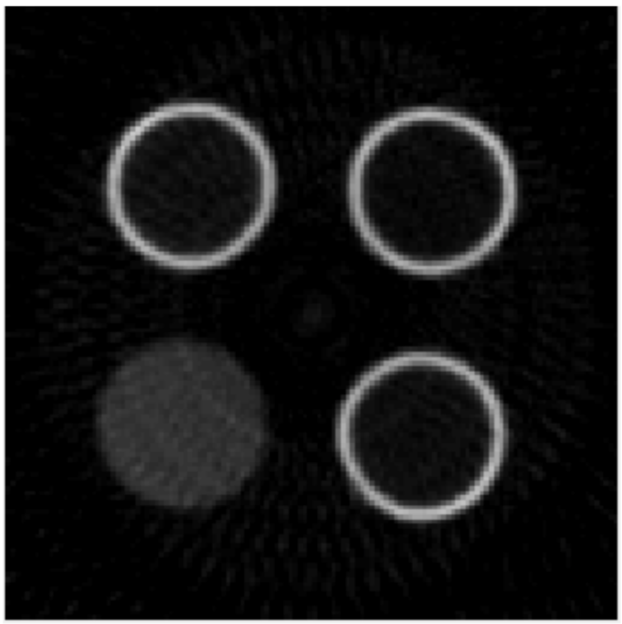
$$I_{\mathrm{ideal}} = Q(I_{\mathrm{meas}})$$

$$S(E) = X_{\mathrm{model}}(E) D_{\mathrm{model}}(E) \exp[-\mathrm{poly}(E)]$$

Two-step recons



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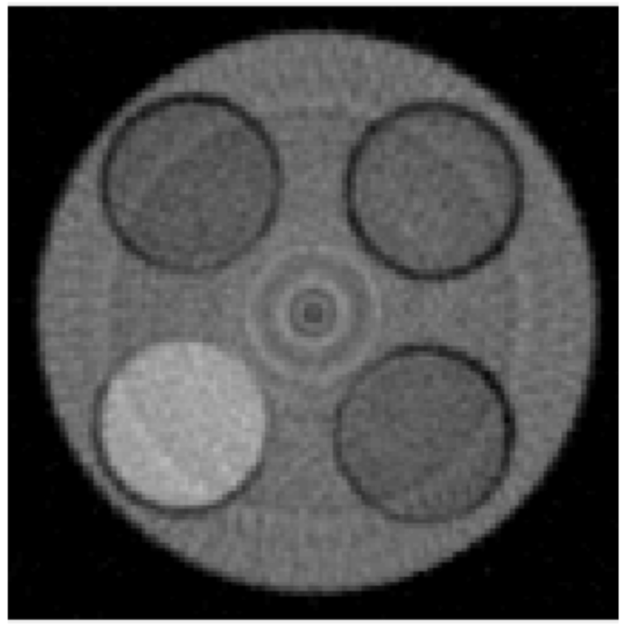

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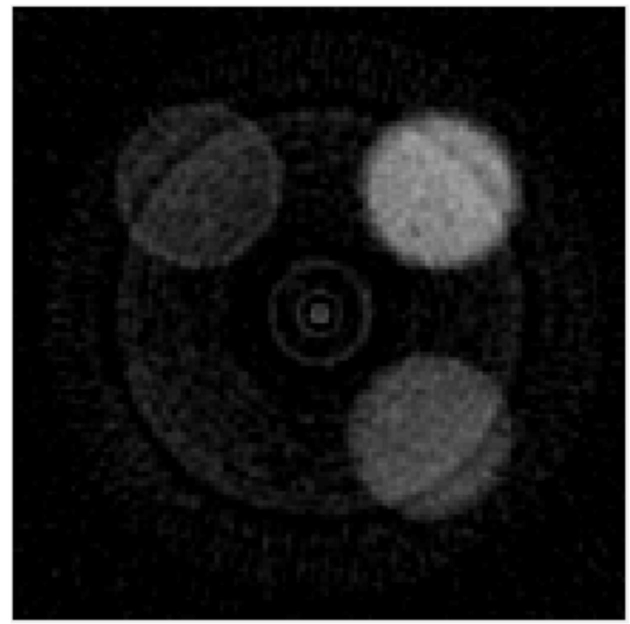
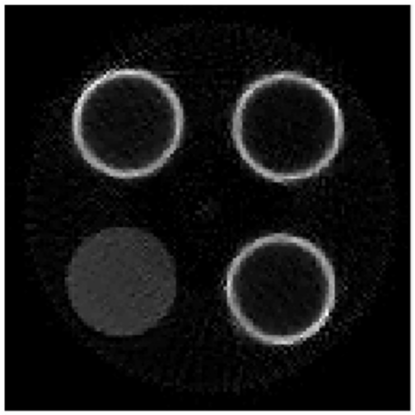


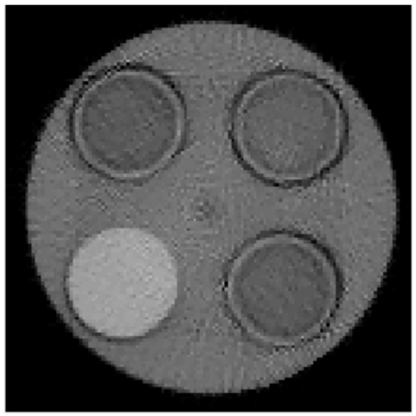
Gd [0, .02]

Spectrum + Dxray model + Nonlinear corr

Images for GTV=2000



Aluminum [0, 0.5]



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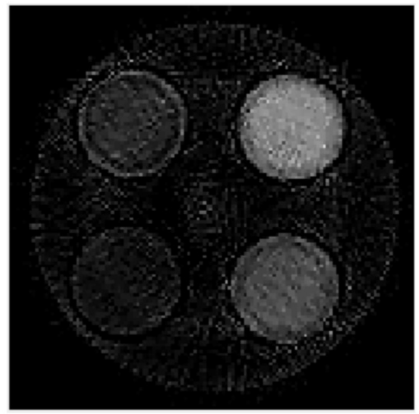
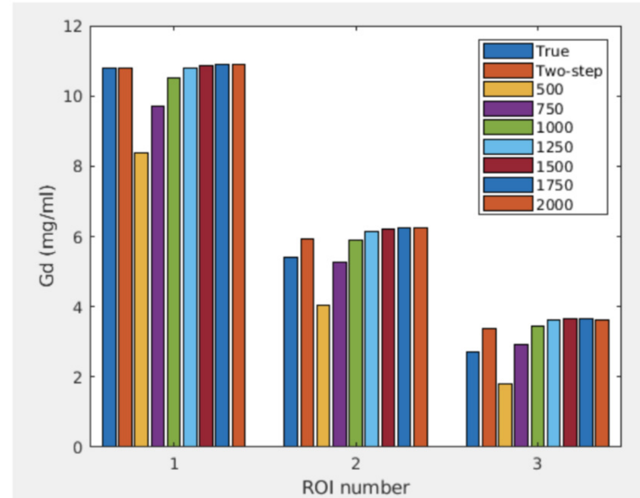
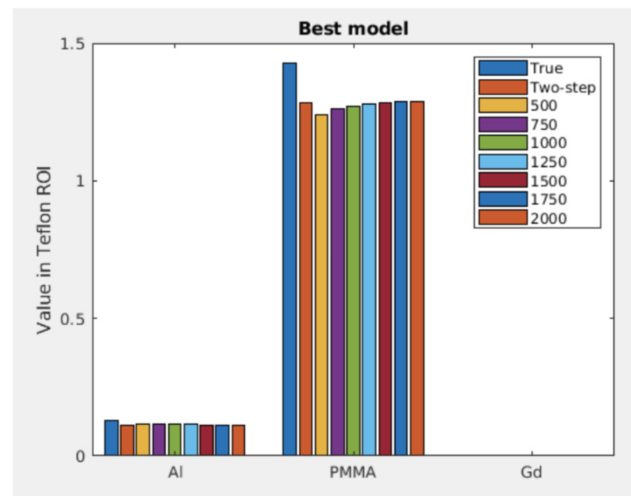


Gd [0, .02]



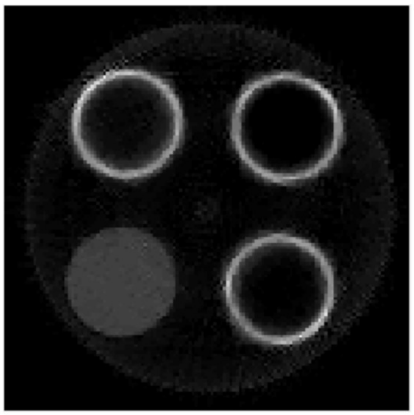


$$I_{\text{ideal}} = Q(I_{\text{meas}})$$

$$S(E) = X_{\text{model}}(E) D_{\text{model}}(E) \exp[-\text{poly}(E)]$$

Spectrum + Nonlinear corr (no det model)

Images for GTV=2000



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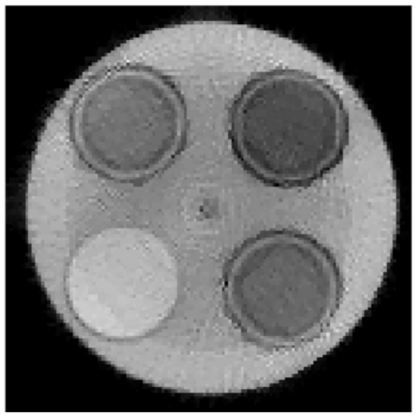

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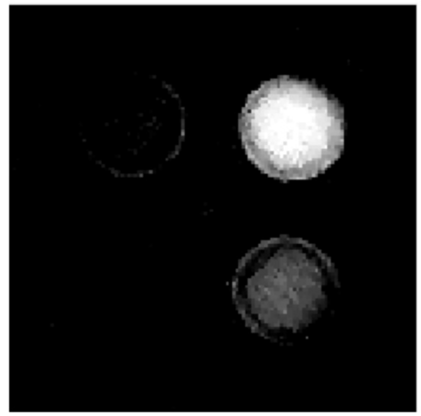
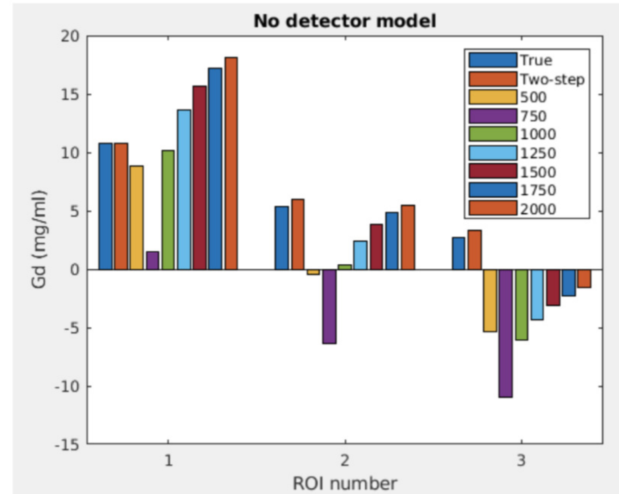
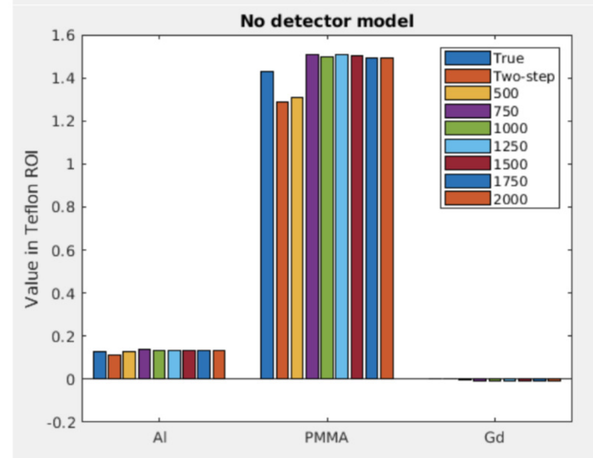


Gd [0, .02]



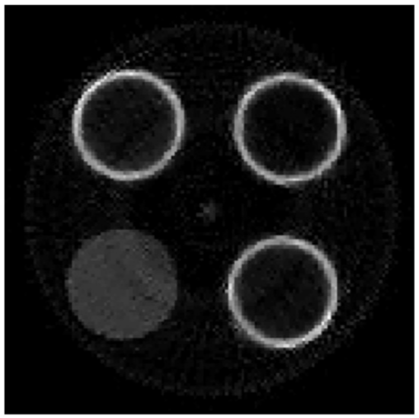


$$I_{\text{ideal}} = Q(I_{\text{meas}})$$

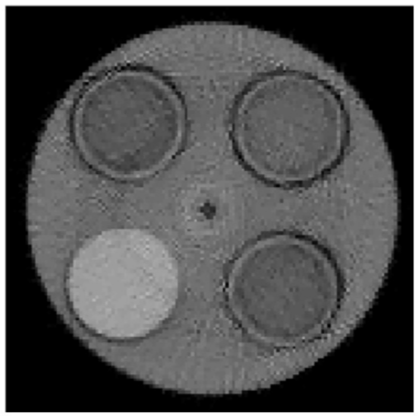
$$S(E) = X_{\text{model}}(E)\exp[-\text{poly}(E)]$$

Dxray model +Nonlinear corr (no source model)

Images for GTV=2000



Aluminum [0, 0.5]



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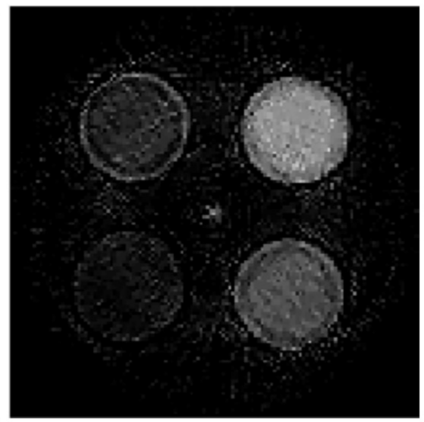
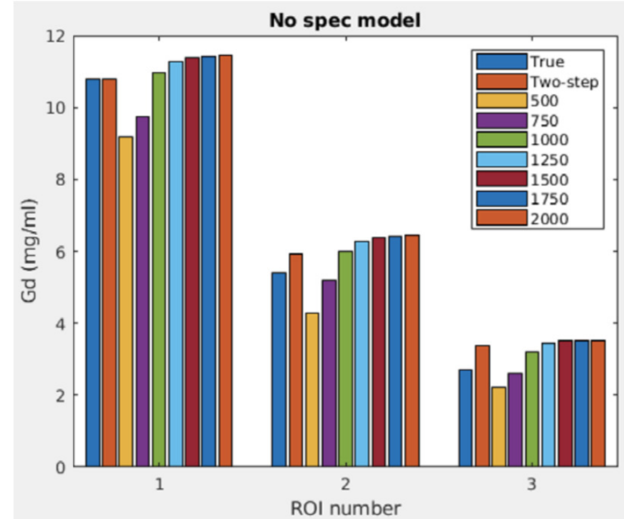
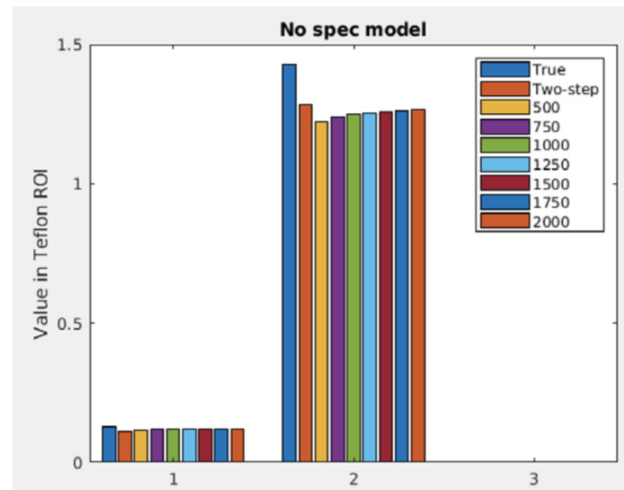


Gd [0, .02]





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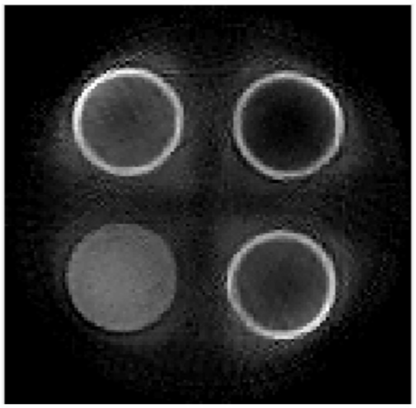
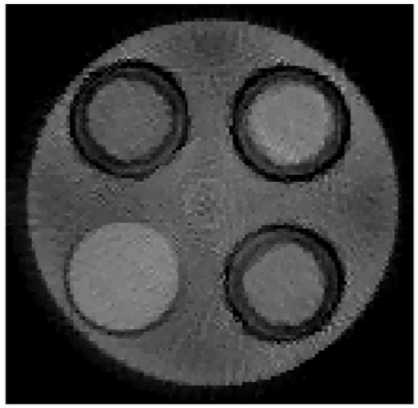
Spectrum + Dxray model (no nonlinear corr)

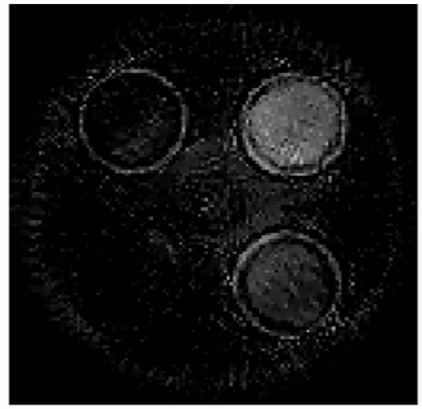
Images for GTV=2000



Aluminum [0, 0.5]



PMMA [0, 2]



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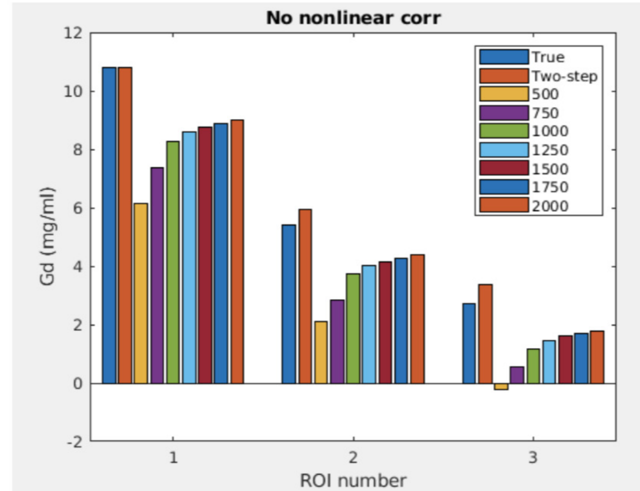

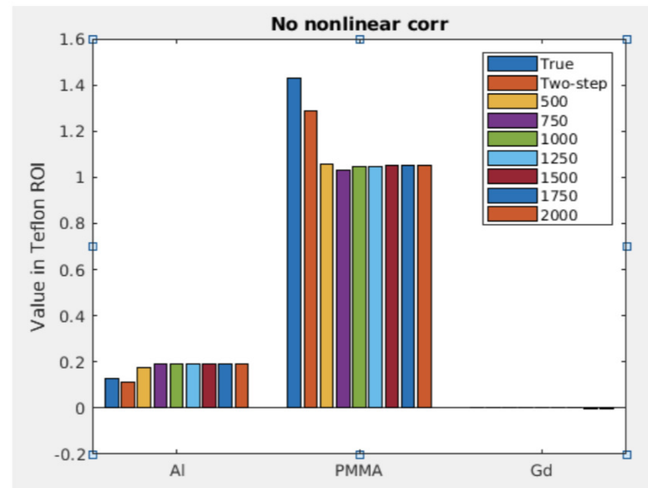

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Future directions

Get better PCD detector!