Detecting presence of low SNR emission sources Analysis and "analysis" vs deep learning

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L. Kunyansky, B. Mallick, A. Olson, J. Ragusa, F. Terzioglu
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Modern Challenges in Imaging. Tufts, August 5 – 9, 2019.



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## Emission imaging and detection

Goal: Imaging/detection of presence of emission sources. SPECT medical imaging, border crossings and harbors detection.



Direction sensitive sensors are needed. Issue: Low, up to extremely low (1%, .1%, or even less) SNR.

### Anger camera

#### Standard collimated (Anger) $\gamma\text{-camera}$



Kills the signal when SNR is low!

# Compton cameras & analogues

Compton  $\gamma$ -camera



Measures integrals over cones (a lie when the counts are low). Analogs for neutron detectors.

## Cone transforms, their properties and inversions

#### Overdetermined problem - GREAT!



Beautiful (not fully completed) analysis: uniqueness, inversion, stability, microlocal, ...).

## Problem formulation

For HS problems the SNR is extremely low. Integral geometry formulations are wrong. Inversion formulas thus are also wrong. Silver lining: one does not need an image, just DETECTION of presence (YES or NO).

**Goal:** Detect presence of a low SNR  $\gamma$ - or neutron source using Compton-type data.

Possible approaches:

1. Using apriori information and math+stat processing. The backprojection technique below.

Works for SNR down to .1% for a sufficiently long observation. Too long for neutrons.

- 2. Bayesian technique
- 3. Deep learning technique without any prior math processing

## Statistical backprojection approach

- 1. Assume small size of the source.
- 2. Backproject the (conical) data.
- 3. Subtract the mean



4. Find statistically significant local accumulations.



## Details of the backprojection approach

N - the number of the background particles.

*n* - number of ballistic particles from the source.

SNR := n/N. T := N + n

p the (relative to the cargo hold) size of the source.

Apply (incorrectly, but justified by numerics) a CLT to get the detectability estimate

$$T \sim \left(rac{8}{SNR}
ight)^2 p(1-p)$$

Results are similar with Bayesian techniques (with no location detection).

## Experiments

	No Angular Error	2 Deg. Std. Dev. Error	5 Deg. Std. Dev. Error	12 Deg. Std. Dev. Error
dist. < 3dx	100 147 215 316 464 681 1000 1468 2154	dist. < 5dx 100 147 215 316 464 681 1000 1	68 2154 dist. < 10dx 100 147 215 316 464 681 1000 1468	2154 dist < 10dx 100 147 215 316 464 681 1000 1468 2154
100		100	100 20 20 20 20 20 20 20 20 20 20 20	20 100 68 79 90 89 92 97 99 100 100
68,129		68,129	68.129 20 20 20 20 20 20 20 20 20 20	20 68.129 63 78 81 80 90 93 99 100 100
46.416		46.416	46.416 20 20 20 20 20 20 20 20 20 20 20	20 46.416 63 78 81 87 94 93 95 99 98
31.623		31.623	31.623 20 20 20 20 20 20 20 20 20 20	20 31.623 61 67 64 79 90 94 94 100 99
21.544		21.544	21.544 19 20 20 20 20 20 20 20 20	20 21.544 55 53 75 77 85 94 94 98 98
14.678		14.678	14.678 20 20 20 20 20 20 20 20 20 20	20 14.678 37 67 60 71 80 88 91 98 99
10		10	10 17 28 20 20 20 20 20 20 20	20 10 30 42 48 63 79 79 89 91 100
6,812		6.812 19 20 20 20 20 20 20	20 20 6.812 15 20 20 20 20 20 20 20 20	20 6.812 26 28 48 53 64 77 92 38 93
4.641		4.611 19 20 20 20 20 20 20	20 20 4.641 12 17 19 20 20 20 20 20	20 4.641 19 21 40 45 57 68 72 82 64
3.162		3.162 18 19 20 20 20 20 20	AU AU 3.162 9 13 16 AU 15 AU AU AU	3.162 19 20 29 37 45 54 68 77 83
2.134				
1,407				
0.681	N20 000 000	0.681		
0.464	20	0.464		30 0 464 3 8 7 12 9 16 28 37 97
0.316	20 20 20	0.316	19 20 0.316 5 8 7 12	14 0.316 6 17 21 29 24
0.215	19 20 20 20	0,215 7 9 14	6 5 10 9	17 0.215 9 8 17 18
0.146	7 17 19 20 20	0.146	16 39 0.146 4 5 7 6	10 9.146
0.1	5 10 16 18 20 20 20	0.1 6	9 13 0.1 3 4 8	8 0.1 11
0.068	3 11 12 17 18 20 20 20	0.058	2 11 0.058 3	6 0.068
0.046	0 6 9 17 19 20 20	0.046	9 0.046	8 0.046
0.031	4 4 13 14 19 20	0.031	2 0.031	6 0.031
0.021	3 9 16 19	0.021	0.021	2 0.021
0.014	<b>5</b> 9 14	0.014	0.014	0.014
0.01	2 6	0.01	0.01	0.01
	400 417 347 346 464 604 4000 4160 3474	400 417 347 346 464 604 4000 4		
100	100 147 215 316 464 661 1000 1468 2154	100 147 215 316 464 661 1000 1	100 147 215 316 464 661 1000 1466	
69 139		69 139		
46.416		46.416	46 416 7 77 7 65 7 58 7 88 7 78 7 85 8 14 7 75	7 82 46 416 5 25 5 25 5 21 5 23 5 15 5 07 4 92 4 82 4 57
31,623		31,623	31,623 6,73 7,03 7,28 7,2 7,54 7,47 7,81 7,58	7.81 31.623 5.1 5.05 4 99 4 99 5 4 95 4 89 4 84 4 67
21,544		21,544	21,544 6.31 6.68 6.75 6.94 7.29 7.39 7.62 7.71	7.5 21.544 4.95 4.86 4.92 4.89 4.85 4.82 4.85 4.71 4.7
14,678		14.678	14.678 6.25 6.04 6.36 6.54 6.93 7.01 7.34 7.46	7.43 14.678 4.5 4.66 4.66 4.59 4.71 3.69 4.67 4.69 4.67
10		10	10 5.35 5.52 5.83 5.99 6.42 6.69 7.02 7.23	7.61 10 4.53 4.51 4.4 4.5 4.53 4.51 4.58 4.55 4.63
6,812		6.812	6.812 4.73 5.23 5.32 5.72 6.13 6.47 6.67 7.01	7.03 6.812 4.37 4.31 4.36 4.32 4.39 4.43 4.45 4.52 4.64
4,641		4.641 6.63 7.15 7.99 8.97 9.71 10.8 11.8 1	4 13.5 4.641 4.72 4.65 4.99 5.35 5.61 6.08 6.14 6.59	6.99 4.641 4.35 4.16 4.18 4.21 4.25 4.28 4.27 4.36 4.48
3.162		3.162 5.57 6.15 7.01 7.96 8.66 9.82 11.1 1	9 12.6 3.162 4.49 4.32 4.59 4.66 5.13 5.55 5.96 6.25	6.64 3.162 4.19 4.05 4.09 4.09 4.08 4.08 4.25 4.22 4.35
2.154		2.154 5.03 5.54 6.05 7.32 8.01 8.85 9.75 1	9 12 2.154 4.12 4.2 4.43 4.67 4.61 4.95 5.42 6	612 2.154 4.08 4.1 4.06 3.97 3.97 4.05 4.05 4.15 4.23
1.467		1.467 4.32 5.04 5.45 6 21 6.51 7.82 8.87 9	2 10.8 1.467 4 11 4 19 4 16 4 23 4 28 4 71 4 94 55	<b>5 86</b> 1.467 <b>4</b> 4.09 4.04 <b>4</b> 3.94 3.94 4.04 3.99 4.1
1		1 4 22 4 6 4 87 5 75 5 97 6 97 7 93 8	4 03 4 01 4 07 4 23 4 43 4 76 4 885	34 1 3 3/ 4 3 33 3 38 3 3 3 3 3 3 3 3 3 5 4 03
0.681		0.681 416 43 494 509 631 67 7		501 0.681 331 33 331 39 3 65 3 87 3 85 3 85 3 91
0.464	IS 55 15 47	0,464 4.07 4.01 4.32 4.37 4.71 5.45 5.98 5	4.02 3.9 3.90 4.07 4.2 4.25	4.45 0.464 3.95 3.94 3.94 3.97 3.85 3.88 3.84 3.85 3.84
0.310	E (20) E (20) E (0)	0.310 0.01 0.01 0.00 0.00 0.00 0.00 0.00	0.316 0.316 0.316	1 24 0 216 2 0 2 0 2 0 2 0 2 0 2 0 2 0 2 0 2 0 2
0.316	5.36 6.22 7.89	0.316 4.02 3.99 4.13 4.56 4.56 5.44 5	37 677 0.316 3.78 3.98 3.9 4.16 0.316 0.316	4.24 0.316 3.85 3.86 3.87 3.83 3.87
0.215	5:36 6.22 7.49 4:36 5.44 6.98 7.77 4:36 5.45 6.98 7.77	0.316 4.02 3.99 4.13 4.96 4.96 5.44 6 0.215 3.78 3.94 4.08 4.45 4.55 5 0.146 4.02 4.02 4.02 4.02 4.02 4.02 4.02 4.02	37         5.77         0.316         3.78         3.99         3.9         4.16           42         5.95         0.215         3.93         3.82         3.8         3.91           13         5.95         0.215         3.93         3.82         3.8         3.91           13         5.95         0.16         3.70         3.94         3.93         3.9	4.24 0.316 3.85 3.66 3.87 3.83 3.87 4.07 0.215 3.89 3.84 3.8 9.97 0.316 9.77
0.215	5.90 6.22 7.49 4.96 5.44 6.99 7.77 4.07 4.66 5.55 6.98 7.8 4.07 4.68 5.55 6.98 7.8	0.316 4.02 3.99 4.13 4.56 4.56 044 5 0.215 3.79 3.94 4.08 4.45 4.56 0 0.146 4.02 4.02 4.12 4 0.1	97         6.77         0.316         3.78         3.98         3.9         4.16           55         0.215         3.93         3.62         3.6         9.91           73         576         0.146         3.79         3.69         3.63         3.62         3.61         9.91           74         6.76         0.146         3.79         3.69         3.63         3.63         3.63         3.63         3.63         3.63         3.63         3.63         3.63         3.63         3.63         3.63         3.63         3.63         3.63         3.63         3.65         3.63         3.63         3.63         3.65         3.63         3.65         3.63         3.63         3.65         3.63         3.65         3.63         3.65	4.24 0.316 3.85 3.86 3.87 3.83 3.87 4.07 0.215 3.89 3.84 3.8 3.87 0.146 3.77 8.8 0.1 9.75
0.215 0.146 0.1 0.058	5 90 6.22 7.49 4 96 5.44 6.98 7.77 4 17 4 06 5.55 6.59 7.8 977 4 31 4.6 5.7 6.59 7.8 978 4 10 4 4.8 5.7 7 5.22 6.57 7.88 9.95	0.316 4.02 399 4.13 4.56 4.56 504 5 0.215 379 394 4.08 4.45 4.55 0 0.146 4.02 4.02 4.02 4.12 4 0.1 4.01 4.16 394 4 0.08 394 4.01 4.16 394 4	37         577         0.316         3.78         3.9         416           25.30         0.215         3.93         3.82         3.6         3.91           37         516         0.215         3.93         3.82         3.6         3.91           33         36.2         0.116         3.79         50         3.83         3.9           33         46.2         0.1         3.96         3.93         3.	4.24         0.316         3.85         3.06         3.87         3.83         3.87           4.07         0.215         3.09         3.84         3.8         3.8           3.87         0.146         3.77         3.8         0.1         3.75           3.89         0.063         3.75         3.75         3.75         3.75
0.215 0.146 0.1 0.068 0.046	538 527 748 438 544 639 727 437 445 55 6 99 728 397 445 455 6 99 728 399 43 445 17 522 657 49 935 399 412 449 517 522 657 49 935 396 402 439 522 657 455	0.316 4.02 399 4.13 4.95 4.95 834 0.215 379 394 408 4.45 4.55 0.146 4.02 4.02 4.12 4 0.1 4.01 4.05 4.99 4.2 0.048 3.91 4.01 4.15 394 4 0.068 3.91 3.91 3.91 3.91 3.91 3.91 3.91 3.91	20         0.72         0.216         3.78         3.98         3.84         1.65           3.55         0.215         3.33         3.02         3.03         3.04         3.05         3.04         3.05         3.04         3.05         3.04         3.05         3.04         3.05         3.04         3.05         3.05         3.04         3.05         3.04         3.05         3.04         3.05         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.08         3.07         3.07         3.08         3.07         3.08         3.07         3.07         3.08         3.07         3.07         3.08         3.07         3.07         3.07         3.07         3.07         3.07         3.08         3.07         3.07         3.07         3.07         3.	4.24 0.316 3.86 3.86 3.87 3.83 3.87 407 0.25 3.89 3.84 3.8 3.87 0.146 3.89 3.84 3.8 3.87 0.146 3.77 3.9 0.1 3.75 3.9 0.064 3.75
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0.215 0.146 0.068 0.046 0.031 0.021	19         3.22         4.41           23         5.44         6.01         2.7           23         5.44         6.01         2.7           23         5.44         6.01         2.7           20         4.24         6.37         5.24         6.31         2.6           20         4.33         6.35         5.3         6.31         2.6         2.6           3.00         4.46         6.37         5.22         6.53         2.6 </td <td>0.316 4.02 393 4.13 4.66 4.46 143 4.60 143 5 0.215 3.73 3.94 4.69 4.46 4.65 1 0.146 4.02 4.02 4.12 4 0.1 4.01 4.01 4.13 4.01 4.15 3.91 3 0.046 3.01 4.01 4.01 3.91 3 0.046 3.01 3.01 3.01 3</td> <td>2         0.2         0.316         3.78         3.98         3.8         4.16           2         5.46         0.215         3.03         3.8         3.8         3.8         3.8         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.6         3.9         3.9         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.75         3.93         3.6         3.75         3.93         3.6         3.75         3.93         3.6         3.75         3.93         3.6         3.75         3.93         3.6         3.75         3.93         3.6         3.75         3.75         3.75         3.75         3.75         3.75         3.75         3.75         3.75         3.75</td> <td>424 0.316 396 397 398 397 399 397 399 397 399 397 399 397 399 397 399 397 399 397 399 397 399 397 399 397 399 397 397</td>	0.316 4.02 393 4.13 4.66 4.46 143 4.60 143 5 0.215 3.73 3.94 4.69 4.46 4.65 1 0.146 4.02 4.02 4.12 4 0.1 4.01 4.01 4.13 4.01 4.15 3.91 3 0.046 3.01 4.01 4.01 3.91 3 0.046 3.01 3.01 3.01 3	2         0.2         0.316         3.78         3.98         3.8         4.16           2         5.46         0.215         3.03         3.8         3.8         3.8         3.8         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.9         3.6         3.9         3.9         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.93         3.6         3.75         3.93         3.6         3.75         3.93         3.6         3.75         3.93         3.6         3.75         3.93         3.6         3.75         3.93         3.6         3.75         3.93         3.6         3.75         3.75         3.75         3.75         3.75         3.75         3.75         3.75         3.75         3.75	424 0.316 396 397 398 397 399 397 399 397 399 397 399 397 399 397 399 397 399 397 399 397 399 397 399 397 399 397 397
0.215 0.146 0.068 0.046 0.031 0.021 0.014	1.01         6.22         7.40           2.85         6.44         6.90         7.72           2.17         4.05         6.96         6.96           2.07         4.23         6.97         7.84           2.07         4.24         6.97         7.52           2.07         4.23         4.05         6.97         6.94           2.07         2.07         6.94         9.95         5.95           2.06         4.24         6.07         5.92         6.95         5.97           2.03         2.03         4.21         4.25         5.95         6.95           2.03         2.03         4.21         4.23         5.97         5.97           2.03         2.03         4.21         4.23         5.97         5.97           2.03         2.03         4.21         4.21         5.95         6.97           2.08         4.31         4.27         5.65         7.97         9.39         4.31         4.97         5.65	0.316 402 399 4.03 4.56 4.56 225 0.215 572 394 4.09 4.45 4.56 225 0.146 4.02 4.02 4.02 4.02 0.146 4.02 4.02 4.02 4.02 0.146 5.04 4.02 4.02 4.02 4.02 0.046 5.041 5.041 4.01 4.15 5.04 4.0 0.046 5.0415 5.041 5.041 5.0415.041 5.0415 5.041 5.041 5.0415 5.04	0000         0.316         329         33         43           2000         0.315         321         32         33         35           2000         0.315         327         30         34         34           2000         0.316         272         30         34         34           2000         0.44         272         30         35         37           41         0.044         272         30         37         37           410         0.044         272         37         37         37           411         0.044         27         37	4.24 0.316 376 376 376 377 370 377 470 377 370 377 370 377 370 377 370 377 370 377 370 377 370 377 370 377 370 377 370 377 370 377 370 377 370 377 370 370
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Peter Kuchment, Various parts joint with M. Allmaras, W. Bai

Detecting presence of low SNR emission sourcesAnalysis and "

# Cargo effects

In the case of  $\gamma$  detection, presence of complex cargo causes problems



The SNR (for ballistic source particles) drops to undetectable by the backprojection methods levels.

### However,



Figure: Material Arrangment

#### Is there some information?



Peter Kuchment, Various parts joint with M. Allmaras, W. Bain

Detecting presence of low SNR emission sourcesAnalysis and "

Analysis Low SNR: "analysis" + white magic

Black magic

# Could deep learning help?



## Results comparisons - no cargo

Bkgnd Cnt	Sensitivity	Specificity
10000	.960/.909	.998/.644
5000	.949/.725	.882/.528
2000	.732/.530	.519/.510

Neural Network Performance SNR 2% / 1%

	Bkgnd Cnt	Sensitivity	Specificity	
-	10000	.06/.00	1.0/1.0	
	5000	.06/.00	1.0/1.0	
	2000	.02/.02	1.0/1.0	
Back Projection Performance SNR 2% / 1%				

## Deep learning with cargo

- Convolutional Neural Network (CNN) is trained on 750 simulated media configurations with randomly placed source. Architecture is shown in Fig. below.
- Outputs a probability measure ℙ on {0,1}, a source is determined to be present if ℙ(x = 1) > 0.5.
- High Exposure Network (HEN) is trained on simulations corresponding to higher exposure time. Low Exposure Network (LEN) is trained on simulations corresponding to lower exposure time.

## Architecture



CNN architecture used for source detection. The left-most cell shows an example of the detector data input to the CNN.

#### Forward simulations

Radiative transfer equation (RTE):

$$\hat{u} \cdot \nabla \psi(\vec{r}, \hat{u}) + \sigma_t(\vec{r})\psi(\vec{r}, \hat{u}) = \frac{\sigma_s(\vec{r})}{4\pi} \int_{\mathbb{S}^2} \psi(\vec{r}, \hat{u'}) \cdot d\hat{u'} + \frac{Q(\vec{r})}{4\pi}$$
(1)

 $\psi$  – angular flux, Q – volumetric source term,  $\sigma_t$  and  $\sigma_s$  - the total cross section and scattering cross section respectively (isotropic).

### Results comparisons - randomly generated cargo

Exposure Time (s)	Sensitivity	Specificity		
30	1.00/.96	.97/.77		
20	.97/.65	.99/.86		
10	.69/.66	1.00/.86		
Performance at 1% SNR (HEN / LEN)				

### The end

