



Tomography With Deep Learning

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August 8, 2019



Outline

Deep Reason

Experimental Results

Explainable Al

Promising Topics

Outline

Deep Reason

Experimental Results

Explainable Al

Promising Topics

Industrial vs Info/Intelligence Techniques



Pre-cursor for Data-driven Recon (2012)

IEEE Trans Med Imaging. Author manuscript; available in PMC 2013 Sep

19.

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<u>IEEE Trans Med Imaging. 2012 Sep; 31(9): 1682–1697.</u>

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PMCID: PMC3777547 NIHMSID: NIHMS509576 PMID: <u>22542666</u>

Low-Dose X-ray CT Reconstruction via Dictionary Learning

Qiong Xu, <u>Hengyong Yu</u>, Senior Member, IEEE,^M <u>Xuanqin Mou</u>,^M <u>Lei Zhang</u>, Member, IEEE, <u>Jiang Hsieh</u>, Senior Member, IEEE, and <u>Ge Wang</u>, Fellow, IEEE

Data-driven Radiomics (Yu & Wang, 2013)



Figure 1. Flowchart of the proposed tensor dictionary and neural network analysis for lung cancer low-dose CT screen to differentiate true/false positive/negative results.

2013-11-18

Application Identifier 14-00706

AAAS Meeting (Feb. 2016)

FEBRUAR	RY 11-15 GL	AAAS 2016 ANNUA .obal science en	L MEETING GAGEMENT	WASHINGTON	I, DC		
REGISTRATION	PROGRAM	FAMILY SCIENCE DAYS	TRAVEL	POSTERS	EXHIBITORS		
PROGRAM - HOME	X-Ra	ay Imaging Innova	tions for Bio	omedicine			
AUTHOR INDEX	Friday, F Coolidge	February 12, 2016: 3:00 PM-4:30 PM e (Marriott Wardman Park)					
Meeting Informati When: February 11 - 15, 2016 Where: Washington, DC	ON In compo organism ray shad role in cl variety o X-ray CT	In computed tomography (i.e., CT scans), X-rays generated in an emission source are used to illuminate an organism, project shadows, and undergo measurement in a detector array. Spatiotemporal multiplexing of X-ray shadows enables computational synthesis of people's internal structures. Today, X-ray CT has a central role in clinical imaging, often as the first and only imaging study before definitive intervention for a wide variety of conditions. More than 100 million CT scans are performed worldwide each year. However, current X-ray CT technology is often insufficient to differentiate benign and malignant etiologies, describe tissue					
	tumor ty drawn co improver these lor informati contrast, scanners and ope	pes and grades, or predict early resp oncerns over potential risk of induced ments in X-ray sources, detectors, ar ng-standing challenges. For example ion content, X-ray gratings extract ref , and contemporary reconstruction m s are being developed to offer superior ration costs. CT scanners also serve	onse to therapy. X-ray I cancer formation. Thi nd reconstruction algor photon-counting deter fractive and elastic sca ethods refine image qu or imaging performanc as a source of big dat	CT involves ionizing rad s symposium highlights ithms that promise to ac ectors add a spectral dim ittering features that imp uality with reduced radia a and minimize production a that can be archived o	diation, which has recent ddress some of hension to the prove soft tissue tion dose. New CT ion, deployment, on the cloud and		

reused for smarter imaging and universal accessibility.

Organizer: Ge Wang, Rensselaer Polytechnic Institute

Co-Organizer: Mannudeep Kalra, Massachusetts General Hospital

AI Talk at AAAS



Roadmap for Deep Recon



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Digital Object Identifier 10.1109/ACCESS.2016.2624938

A Perspective on Deep Imaging

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ABSTRACT The combination of tomographic imaging and deep learning, or machine learning in general, promises to empower not only image analysis but also image reconstruction. The latter aspect is considered in this perspective article with an emphasis on medical imaging to develop a new generation of image reconstruction theories and techniques. This direction might lead to intelligent utilization of domain knowledge from big data, innovative approaches for image reconstruction, and superior performance in clinical and preclinical applications. To realize the full impact of machine learning for tomographic imaging, major theoretical, technical and translational efforts are immediately needed.

Acknowledgment





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Smart Precision Imaging/Medicine



FIGURE 2. Big picture of deep imaging – A full fusion of medical imaging and deep learning. A high likelihood is that the direct paths from data to features and actions may need an intermediate layer essentially equivalent to a reconstructed/processed image.

IEEE Trans. Medical Imaging Special Issue

IEEE TRANSACTIONS ON MEDICAL IMAGING, VOL. 37, NO. 6, JUNE 2018

1289

Image Reconstruction Is a New Frontier of Machine Learning

Ge Wang[®], *Fellow, IEEE*, Jong Chu Ye[®], *Senior Member, IEEE*, Klaus Mueller[®], *Senior Member, IEEE*, and Jeffrey A. Fessler[®], *Fellow, IEEE*

I. INTRODUCTION

VER the past several years, machine learning, or more generally artificial intelligence, has generated overwhelming research interest and attracted unprecedented public attention. As tomographic imaging researchers, we share the excitement from our imaging perspective [item 1) in the Appendix], and organized this special issue dedicated to the theme of "Machine learning for image reconstruction." This special issue is a sister issue of the special issue published in May 2016 of this journal with the theme "Deep learning in medical imaging" [item 2) in the Appendix]. While the previous special issue targeted medical image processing/analysis, this special issue focuses on data-driven tomographic reconstruction. These two special issues are highly complementary, since image reconstruction and image analysis are two of the main pillars for medical imaging. Together we cover the whole workflow of medical imaging: from tomographic raw data/features to reconstructed images and then extracted diagnostic features/readings.



Strong Momentum of AI/ML



Figure 5: Web of Knowledge search, with "deep learning", "medical", and "imaging" all as the topic terms (Left), and with "deep learning" in the article title (Right). Data collected on July 11, 2019.

Major Players of Al/ML

Showing 46,802 records for TOPIC: (deep learning)	Citation report feature not available		
Visualization Treemap Number of results 10	•	🕹 Download	d Hide
13,471 USA	3,082 England	1,982 AUSTRALIA JAPAN	7
12,784 PEOPLES R CHINA	2,206 germany	1,850 South Korea	1,416 SPAIN
	1,985 canada	1,642 INDIA	

Book on AI/ML Tomography

A learning revolution

The groundwork for machine learning was laid down in the middle of last century. But increasingly powerful computers - harnessed to algorithms refined over the past decade - are driving an explosion of applications in everything from medical physics to materials, as Marric Stephens discovers

When your bank calls to ask about a suspiciously and autonomously. As a subset of the more general Marrie Stephens is large purchase made on your credit card at a strange field of artificial intelligence (AI), machine-learning afreelancescience time, it's unlikely that a kindly member of staff has techniques can be applied wherever there are large writerbased in personally been combing through your account. and complex data sets that can be mined for associa- Bristol, UK, e-mail Instead, it's more likely that a machine has learned tions between inputs and outputs. In the case of your mloyd.stephens@ what sort of behaviours to associate with criminal bank, the algorithm will have analysed a vast pool gmail.com activity - and that it's spotted something unexpected of both legitimate and illegitimate transactions to on your statement. Silently and efficiently, the bank's produce an output ("suspected fraud") from a given computer has been using algorithms to watch over input ("high-value order placed at 3 a.m."). But machine learning isn't just used in finance. It's your account for signs of theft.

Digital technologies: Machine learning

Monitoring credit cards in this way is an exam- being applied in many fields, from healthcare and ple of "machine learning" - the process by which transport to the criminal-justice system. Indeed, Ge a computer system, trained on a given set of exam- Wang - a biomedical engineer from the Rensselaer ples, develops the ability to perform a task flexibly Polytechnic Institute in the US who is one of those



• A new IOP Publishing ebook Machine Learning for Tomographic Imaging by Ge Wang, Yi Zhang, Xiaojing Ye and Xuanqin Mou will be published later this year.

Physics World March 2019

https://physicsworld.com/a/a-machine-learning-revolution

phy sics world.co

AI/ML Tomography Book from IOP Publishing

Image Analysis, One of Successful Applications of Artificial Intelligence & Machine Learning

Tomographic Data Acquisition

> Data As Tomographic Features



Theme of This Book: Tomographic Image Reconstruction

Reconstructe

Õ

Image



Coauthors of the Book:

Yi Zhang, Xiaojing Yu, Xuanqin Mou

Outline of Our Book



New Algorithmic Category

Table 1: Three types of tomographic image reconstruction algorithms in an over-simplified comparison (penalization of image reconstruction and topology of network architecture can be complicated.

Category	Form	Knowledge	Input	Quality	Speed
Analytic Recon	f = O[p]	Idealized Model, Without Noise	High SNR, Compute	High	High
Iterative Recon	$f^{(k)} = O[p, f^{(k-1)}]$	Physical Model, Image Prior	Low in Various Ways	Decent	Low
Deep Recon	$f = O_{\alpha_{\rm N}} \dots O_{\alpha_{\rm l}}[p]$	Model, Prior, Big Training Data	Poor, Incomplete	Superior, Task-specific	High

Image Reconstruction: From Sparsity to Data-adaptive Methods and Machine Learning

Saiprasad Ravishankar, Member, IEEE, Jong Chul Ye, Senior Member, IEEE, and Jeffrey A. Fessler, Fellow, IEEE

Abstract—The field of medical image reconstruction has seen roughly four types of methods. The first type tended to be analytical methods, such as filtered back-projection (FBP) for X-ray computed tomography (CT) and the inverse Fourier transform for magnetic resonance imaging (MRI), based on simple mathematical models for the imaging systems. These methods are typically fast, but have suboptimal properties such as poor resolution-noise trade-off for CT. A second type is iterative reconstruction methods based on more complete models for the imaging system physics and, where appropriate, models for the sensor statistics. These iterative methods improved image quality by reducing noise and artifacts. The FDA-approved methods among these have been based on relatively simple regularization models. A third type of methods has been designed to accommodate modified data acquisition methods, such as reduced sampling in MRI and CT to reduce scan time or radiation dose. These methods typically involve mathematical image models involving assumptions such as sparsity or lowrank. A fourth type of methods replaces mathematically designed models of signals and systems with data-driven or adaptive models inspired by the field of machine learning. This paper reviews the progress in medical image reconstruction methods with focus on the two most recent trends: methods based on sparsity or low-rank models, and data-driven methods based on machine learning techniques.

acquisition time) or low-dose or sparse-view data in CT (reducing patient radiation exposure) has been a popular area of research and holds high value in improving clinical throughput and patient experience. This paper reviews some of the major recent advances in the field of image reconstruction, focusing on methods that use sparsity, low-rankness, and machine learning. We focus partly on PET, SPECT, CT, and MRI examples, but the general methods can be useful for other modalities, both medical and non-medical. Other papers in this issue emphasize other modalities.

A. Types of Image Reconstruction Methods

Image reconstruction methods have undergone significant advances over the past few decades, with different paths for various modalities. These advances can be broadly grouped in four categories of methods. The first category consists of analytical and algebraic methods. These methods include the classical filtered back-projection (FBP) methods for Xray CT (e.g., Feldkamp-Davis-Kress or FDK method [1]) and the inverse Fast Fourier transform and extensions such as

Progress Through Questioning

o Analytic Reconstruction

Given a finite number of projections, the tomographic reconstruction is not uniquely determined (ghosts).

o Statistical Reconstruction

A reconstructed image is strongly influenced by the penalty term, and what you see is what you want to see!

Compressed Sensing

There is a chance that a sparse solution is not the truth. For example, physiological texture and/or pathological plaques incorrectly eliminated

o Machine Learning

No Maxwell equations for machine learning, and a neural network as a black box is trained to work with big data through parameter adjustment

Machine Learning to Dominate

In Principle, Machine Learning (ML) Can Outperform Analytic Reconstruction (AR), Iterative Reconstruction (IR) / Compressed Sensing (CS)

AR/IR/CS Used as

- Component (Such as in the "LEARN" Network)
- Baseline (Such as for Image Denoising)
- IR/CS Enhanced/Replaced by Neural Networks (As Extensive Priors & Powerful Non-learning Mapping, Driven by Big Data)

Superiority Principle



Can New Dog be Better in Old Tricks



Spiral Single-slice CT



Theoretical Superiority

"For a given X-ray dose, helical CT allows substantially better longitudinal resolution than conventional CT due to its inherent retrospective reconstruction capability."



Wang and Vannier Medical Physics 21:429-433, 1994

Wang, Brink, Vannier Medical Physics 21:753-754, 1994



Retrospective Reconstruction



Incremental (Left) vs Spiral (Right) Scans Define Imaging Planes Differently. The Former Specifies Imaging Planes Physically/Prospectively, while the Latter Does so Computationally/Retrospectively.

Superior Detectability



Retrospective Reconstruction Gives Better Lesion Detectability If There Are Sufficiently Many Slices Reconstructed!

Top Level Comparison in LDCT Performance



machine intelligence

Article | Published: 10 June 2019

Competitive performance of a modularized deep neural network compared to commercial algorithms for low-dose CT image reconstruction

Hongming Shan, Atul Padole, Fatemeh Homayounieh, Uwe Kruger, Ruhani Doda Khera, Chayanin Nitiwarangkul, Mannudeep K. Kalra[™] & Ge Wang[™]

Nature Machine Intelligence 1, 269–276 (2019) | Download Citation 🕹

IR Methods vs MAP DL for Low-dose CT

Commercial Iterative Recon (IR) Algorithms in This Study





Our MAP Network-based Deep Learning (DL) with Optimized Depth



Best versus Best



Best Deep Recon (DR) vs Best Iterative Recon (IR) Algorithms in This Study across Vendors, Body Regions, & Readers

Outline

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Experimental Results

Explainable AI

Promising Topics

Low-dose CT Denoising: FBP + Network



IEEE Trans. Medical Imaging 37:1522-1534, 2018 (for details, see <u>https://arxiv.org/abs/1802.05656</u> or <u>https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=8353466</u>)

GE Medical CT Denoising (RSNA'18)



0.625 mm FBP

0.625 mm ASiR-V 50%

0.625 mm TrueFidelity image

Low-cost CT Deblurring: GAN–CIRCLE



You CY, Zhang Y, Zhang XL, Ju SH, Zhang ZY, Zhao Z, Cong WX, Saha PK, Wang G: CT Super-resolution GAN Constrained by the Identical, Residual, and Cycle Learning Ensemble (GAN-CIRCLE), arXiv, Aug. 2018



Super-resolution for Bone CT



Human Distal Tibia Dataset:

- Low Resolution CT: Siemens FLASH
- Super-resolution CT: GAN-O
- High Resolution CT: Siemens FORCE

With Univ. of Iowa, Dr. Saha's Group

Ensemble Learning for MRI Super-resolution

arXiv.org > eess > arXiv:1907.03063

Electrical Engineering and Systems Science > Image and Video Processing

MRI Super-Resolution with Ensemble Learning and Complementary Priors


Sparse-data CT De-artifacts: "LEARN"



Chen H, Zhang Y, Chen YJ, Zhang WH, Sun HQ, Lv Y, Liao PX, Zhou JL, Wang G:

LEARN: Learned Experts' Assessment-based Reconstruction Network for Sparse-data CT. IEEE Trans. Medical Imaging, June 2018

iCT Network



Fig. 1. Architecture of iCT-Net. The proposed deep neural network consists of a total of 12 layers (L1-L12). The L11 layer is a frozen layer, which means that parameters in this layer are not updated in the training process. Both linear and nonlinear activations are used as indicated in the graphics. S_{λ} is a hard thresholding activation function defined in Eq. (2).

Li, Yinsheng & Ii, ke & Zhang, Chengzhu & C. Montoya, Juan & Chen, Guang-Hong. (2019). Learning to Reconstruct Computed Tomography (CT) Images Directly from Sinogram Data under A Variety of Data Acquisition Conditions. IEEE Transactions on Medical Imaging. PP. 10.1109/TMI.2019.2910760.

Exterior Tomography





IEEE Access

Received August 19, 2016, accepted August 29, 2016, date of publication September 13, 2016, date of current version October 6, 2016. Digital Object Identifier 10.1109/ACCESS.2016.2608621

Metal Artifact Reduction in CT: Where Are We After Four Decades?

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Corresponding authors: B. De Man (deman@ge.com) and G. Wang (ge-wang@ieee.org)

Metal Artifact Reduction: Ensemble Learning



A female with diffused subarachnoid hemorrhage (in the red square). CT angiography demonstrated a left middle cerebral artery aneurysm, which was clipped. The display window is [-100 200] HU.

Yanbo Zhang and Hengyong Yu; A convolutional neural network base metal artifact reduction in x-ray computed tomography. IEEE Transactions on Medical Imaging, 37(6):1370-1381, 2018.

Dual-stream Data Processing Flowchart

Phantom derived from clinical images with metal added



Interior Tomography







Left: The conventional FBP reconstruction from a complete dataset of a sheep chest CT scan (the white circle identifies our selected ROI).

Right: Our TV-minimization-based interior reconstruction from truncated projections associated with x-rays only through the ROI. The sheep scan was done by Dr. Eric Hoffman, University of Iowa, Iowa City, USA

Wang G, Yu HY: Can interior tomography outperform lambda tomography? Proc. Natl. Acad. Sci. USA, 2010. 107(22): p. E92-3

iCT Reconstruction



Fig. 10. The iCT-Net reconstruction results of real human subject data acquired in an abdomen-pelvis scan protocol with the short-scan angular range, super-short scan angular range and interior problem. Dense view reconstruction results are presented in the 2nd column and sparse view reconstruction results are presented in the 3rd column. The corresponding reference images were generated by applying a standard FBP method with a Ram-Lak filter at full FOV ($\emptyset = 50$ cm) from 644 view angles densely sampled across a short-scan angular range. Note that the central portion of the FBP reconstruction without truncation was cropped to generate the reference image for the interior problem with the truncated FOV ($\emptyset = 12.5$ cm).



FBP2ADMIRE: Computational Acceleration



Yanbo Zhang, Robert MacDougall and Hengyong Yu; Convolutional neural network based CT image post-processing from FBP to ADMIRE. Proceedings of the 5th CT Meeting, pp.411-414, 2018

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Interpretability Problem



Basic Types of Neurons



Bipolar Unipolar Multipolar Pyrimidal (Interneuron) (Sensory Neuron) (Motoneuron) Cell

New Type of Neurons

Numerical Methods in Biomedical Engineering

Explore this journal >

RESEARCH ARTICLE

A new type of neurons for machine learning

Fenglei Fan, Wenxiang Cong, Ge Wang 🗠

First published: 15 September 2017 Full publication history

DOI: 10.1002/cnm.2920 View/save citation

Cited by (CrossRef): 0 articles 4 Check for updates





Early View



Browse Early View Articles Online Version of Record published before inclusion in an issue

XOR Gate



XOR-like function by the proposed 2nd order neuron after 100 iterations

Double Spirals

Int J Numer Method Biomed Eng. 2018 May;34(5):e2956. doi: 10.1002/cnm.2956. Epub 2018 Feb 6.

Generalized backpropagation algorithm for training second-order neural networks.

Fan F¹, Cong W¹, Wang G¹.



Lang and Witbrock reported that the standard backpropagation network cannot classify such spirals, and made a 2-5-5-5-1 network with shortcuts to solve this problem. With 2nd order neurons, we can do so with a simpler network without any shortcut.

KJ Lang, MJ Witbrock: Learning to Tell Two Spirals Apart. In Proceedings of the 1988 Connectionist Models Summer School. San Mateo, CA, 52-59, 1989

Sorting CT & MRI Images

Int J Numer Method Biomed Eng. 2018 May;34(5):e2956. doi: 10.1002/cnm.2956. Epub 2018 Feb 6.

Generalized backpropagation algorithm for training second-order neural networks.



Fuzzy Logic Interpretation

Fuzzy Logic Interpretation of Artificial Neural Networks

Fenglei Fan, Student Member, IEEE, Ge Wang, Fellow, IEEE

Abstract — Over past several years, deep learning has achieved huge successes in various applications. However, such a datadriven approach is often criticized for lack of interpretability. Recently, we proposed artificial quadratic neural networks consisting of second-order neurons in potentially many layers. In each second-order neuron, a quadratic function is used in the place of the inner product in a traditional neuron, and then undergoes a nonlinear activation. With a single second-order neuron, any fuzzy logic operation, such as XOR, can be implemented. In this sense, any deep network constructed with quadratic neurons can be interpreted as a deep fuzzy logic system. Since traditional neural networks and second-order counterparts can represent each other and fuzzy logic operations are naturally implemented in secondorder neural networks, it is plausible to explain how a deep neural network works with a second-order network as the system model. In this paper, we generalize and categorize fuzzy logic operations implementable with individual second-order neurons, and then perform statistical/information theoretic analyses of exemplary quadratic neural networks.

were identified [8]. However, these results do not reveal the inner working of a network, such as what and how features are extracted and propagated between layers. Gu *et al.* 2017 [9] offered an elegant explanation of the adversarial mechanism of GAN from the viewpoint of optimal mass transportation. Dong *et al.* 2017 [10] established a correspondence between deep networks and numerical ordinary differential equations to guide the structural design of a network with skip connections.

Instead of handling with existing models directly, researchers also tried to find the models that are more interpretable. For example, Wu *et al.* [11] utilized tree regularization to optimize a deep model with more interpretability. Fan [12] proposed a generalized hamming network based on the fact that neurons calculate generalized hamming distance when a bias is adopted. Albeit novel and interesting models developed in these pilot studies, these arts do not reveal the key mechanism based on which the existing models are so successful.

Deep Fuzzy Logic System



Deep Fuzzy Features



Fig. 3. Second-order network for recognition of Arabic digits from the MNIST dataset. The neurons in the "convolutional" layers are colored and inflated to demonstrate the types and frequencies of quadratic operations.

Software Engineering Principles

• Divide-Conquer

Modularity, Abstraction, Cohesion & Coupling

- Formality
- Generality
- Scalability
- Reliability
- Adaptability

Iterative Refinement, Anticipation of Change



Approximation with Width



Width- versus Depth-Efficiency

arXiv.org > cs > arXiv:1709.02540

Computer Science > Machine Learning

The Expressive Power of Neural Networks: A View from the Width

Zhou Lu, Hongming Pu, Feicheng Wang, Zhiqiang Hu, Liwei Wang

(Submitted on 8 Sep 2017 (v1), last revised 1 Nov 2017 (this version, v3))

Theorem 4. Let n be the input dimension. For any integer $k \ge n + 4$, there exists $F_{\mathscr{A}} : \mathbb{R}^n \to \mathbb{R}$ represented by a ReLU neural network \mathscr{A} with width $d_m = 2k^2$ and depth h = 3, such that for any constant b > 0, there exists $\epsilon > 0$ and for any function $F_{\mathscr{B}} : \mathbb{R}^n \to \mathbb{R}$ represented by ReLU neural network \mathscr{B} whose parameters are bounded in [-b, b] with width $d_m \le k^{3/2}$ and depth $h \le k + 2$, the following inequality holds:

$$\int_{\mathbb{R}^n} \left(F_{\mathscr{A}} - F_{\mathscr{B}} \right)^2 \mathrm{d}x \ge \epsilon.$$
(6)

Abstract

The expressive power of neural networks is important for understanding deep learning. Most existing works consider this problem from the view of the depth of a network. In this paper, we study how width affects the expressiveness of neural networks. Classical results state that *depth-bounded* (e.g. depth-2) networks with suitable activation functions are universal approximators. We show a universal approximation theorem for width-bounded ReLU networks: width-(n + 4) ReLU networks, where n is the input dimension, are universal approximators. Moreover, except for a measure zero set, all functions cannot be approximated by width-nReLU networks, which exhibits a phase transition. Several recent works demonstrate the benefits of depth by proving the depth-efficiency of neural networks. That is, there are classes of deep networks which cannot be realized by any shallow network whose size is no more than an *exponential* bound. Here we pose the dual question on the width-efficiency of ReLU networks: Are there wide networks that cannot be realized by narrow networks whose size is not substantially larger? We show that there exist classes of wide networks which cannot be realized by any narrow network whose depth is no more than a *polynomial* bound. On the other hand, we demonstrate by extensive experiments that narrow networks whose size exceed the polynomial bound by a constant factor can approximate wide and shallow network with high accuracy. Our results provide more comprehensive evidence that depth may be more effective than width for the expressiveness of ReLU networks.

Width: n+4 & Depth: Deep

x_1 x_2 $\bullet \bullet \bullet$ x_n	
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$L_{1} = ((1 - (x_{1} - b_{1} + \delta (a_{1} - b_{1}))) - (1 - (x_{1} - a_{1})^{+} / \delta)^{+}$)+/δ)+)+
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$\begin{array}{c c} (X_1+N)^+ \\ \hline & (X_2+N)^+ \\ \hline & \bullet \bullet \bullet \\ \hline & (X_n+N)^+ \\ \hline & K_1 \\ \hline & 0 \\ \hline \hline & 0 \\ \hline \hline \hline & 0 \\ \hline \hline \hline & 0 \\ \hline \hline \hline \hline & 0 \\ \hline \hline$	
Figure 1: One block to simulate the indicator function on $[a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_n, b_n]$. For k from 1 to n, we "chop" two sides in the kth dimension, and for every k the "chopping" process is completed within a 4-layer sub-network as we show in Figure 1. It is stored in the (n+3)th node as L_n in the last layer of \mathscr{A} . We then use a single layer to record it in the (n+1)th or the (n+2)th node, and reset the last two nodes to zero. Now the network is ready to simulate another (n+1)-dimensional	

cube.

JMLR: Workshop and Conference Proceedings vol 49:1–34, 2016 The Power of Depth for Feedforward Neural Networks

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Abstract

We show that there is a simple (approximately radial) function on \mathbb{R}^d , expressible by a small 3-layer feedforward neural networks, which cannot be approximated by any 2-layer network, to more than a certain constant accuracy, unless its width is exponential in the dimension. The result holds for virtually all known activation functions, including rectified linear units, sigmoids and thresholds, and formally demonstrates that depth – even if increased by 1 – can be exponentially more valuable than width for standard feedforward neural networks. Moreover, compared to related results in the context of Boolean functions, our result requires fewer assumptions, and the proof techniques and construction are very different.

Theorem 1. Suppose the activation function $\sigma(\cdot)$ satisfies assumption I with constant c_{σ} , as well as assumption 2. Then there exist universal constants c, C > 0 such that the following holds: For every dimension d > C, there is a probability measure μ on \mathbb{R}^d and a function $g : \mathbb{R}^d \to \mathbb{R}$ with the following properties:

- 1. g is bounded in [-2, +2], supported on $\{\mathbf{x} : \|\mathbf{x}\| \le C\sqrt{d}\}$, and expressible by a 3-layer network of width $Cc_{\sigma}d^{19/4}$.
- 2. Every function f, expressed by a 2-layer network of width at most ce^{cd} , satisfies

 $\mathbb{E}_{\mathbf{x}\sim\mu}\left(f(\mathbf{x}) - g(\mathbf{x})\right)^2 \ge c.$



Quadratic Neural Network for Human-like Learning

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Abstract

Deep learning is the mainstream of machine learning that concerns with algorithms inspired by the functions of the human brain.

Inspired by the diversity of biologic neurons, our group recently proposed quadratic neurons [1] by replacing the inner product in conventional neurons with a quadratic operation of input data, thereby enhancing the capability of the individual neuron. For instance, even a single quadratic neuron can realize the XOR logic. Along this direction, we are motivated to unlock the power of quadratic neurons in representative network architectures, towards human-like learning in the form of quadratic deep learning.

Introduction

<u>*Quadratic neuron*</u> is upgraded the conventional neuron, which integrates input data into an inner product, into the quadratic neuron that processes the n-dimension inputs as follows:

$$h(\mathbf{x}) = \left(\sum_{i=1}^{n} w_{ir} x_i + b_r\right) \left(\sum_{i=1}^{n} w_{ig} x_i + b_g\right) + \sum_{i=1}^{n} w_{ib} x_i^2 + c$$
$$= \left(w_r x^T + b_r\right) \left(w_g x^T + b_g\right) + w_b \left(x^2\right)^T + c$$

where only 3n parameters are used.



Algebraic Structure

We theoretically demonstrated the strength of quadratic networks [2] in the unique functional representation – a univariate polynomial of degree N can be expressed as $P_N(x) = C \prod_{i=1}^{l_1} (x - x_i) \prod_{i=1}^{l_2} (x^2 + a_i x + b_i)$.



Fig.2. Quadratic network approximates a univariate polynomial according to the Algebraic Fundamental Theorem.

<u>Deliverable</u>: The quadratic deep learning model will empower us to build more powerful Al tools that can help solve complex tasks.

Model Efficiency

With a huge market potential, scaling deep learning to mobile/wearable apps has a major traction. We demonstrated merits of quadratic networks in terms of model efficiency [2].

<u>Theorem</u>: Given the network with only one hidden layer, there exists a function that a quadratic network can approximate it with a polynomial number of neurons but a conventional network can only do the same-level job with

exponentially more neurons.

Proof: The key is to utilize the properties of the Fourier transform of inner products.

<u>Deliverable</u>: Real-time on-site Al modules will be valuable in wearable medical devices.

Fig.3 Heuristics of proof



Lack of the interpretability has become a primary obstacle to the wide-spread translation and development of deep learning. We propose to interpret neural networks from the perspective of engineering. We consider a deep neural network as an integrated system of fuzzy logic gates. Each quadratic module can be topologically characterized by its eigen spectrum [3].



<u>Deliverable</u>: Pushing explainable AI into IBM medical products so that trust is gained from patients and other customers.

Future Directions

- Modularize important quadratic networks
- · Hybrid networks with more bio-plausibility [4]
- · Develop a fuzzy theory of quadratic networks



Reference

[1] Fan F, et al., IJN /IBE, 2018, 34.2, e2920. [2] Fan F, Wang G. : rXiv preprint arXiv:1808.00098. 2018 Jul 31. [3] Fan F, Wang G. : rXiv preprint arXiv:1807.03215. 2018 Jul 4. [4] Krotov D, Hopfield J. arXiv preprint arXiv:1806.10181. 2018 Jun 26.

Universal Approximation Quadratically

arXiv.org > cs > arXiv:1808.00098

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Computer Science > Machine Learning

Universal Approximation with Quadratic Deep Networks

Fenglei Fan, Ge Wang

(Submitted on 31 Jul 2018 (v1), last revised 9 Oct 2018 (this version, v2))

Recently, deep learning has been playing a central role in machine learning research and applications. Since AlexNet, increasingly more advanced networks have achieved state-of-the-art performance in computer vision, speech recognition, language processing, game playing, medical imaging, and so on. In our previous studies, we proposed quadratic/second-order neurons and deep quadratic neural networks. In a quadratic neuron, the inner product of a vector of data and the corresponding weights in a conventional neuron is replaced with a quadratic function. The resultant second-order neuron enjoys an enhanced expressive capability over the conventional neuron. However, how quadratic neurons improve the expressing capability of a deep quadratic network has not been studied up to now, preferably in relation to that of a conventional neural network. In this paper, we ask three basic questions regarding the expressive capability of a quadratic network: (1) for the one-hidden-layer network structure, is there any function that a quadratic network can approximate much more efficiently than a conventional neurons? (2) for the same multi-layer network structure, is there any function that can be expressed by a quadratic network but cannot be expressed with conventional neurons in the same structure? (3) Does a quadratic network give a new insight into universal approximation? Our main contributions are the three theorems shedding light upon these three questions and demonstrating the merits of a quadratic network in terms of expressive efficiency, unique capability, and compact architecture respectively.

Univariate Polynomial of Order N



Algebraic Fundamental Theorem



Particle/Factor Mathematics



Kolmogorov

$$f(\mathbf{x})=f(x_1,\ldots,x_n)=\sum_{q=0}^{2n}\Phi_q\left(\sum_{p=1}^n\phi_{q,p}(x_p)
ight)$$

Also, aided by the concept of partially separable functions, the complexity of the quadratic network can be further reduced, such as in the case of computing an L^{th} separable function. By the L^{th} separable function, we mean that $f(x_1, ..., x_n)$ is L^{th} separable defined as follows:

$$f(x_1, ..., x_n) = \sum_{l=1}^{L} \prod_{i=1}^{n} \phi_{li}(x_i).$$

In practice, almost all continuous functions can be represented as L^{th} separable functions, which are of low ranks at the same time.

Outline

Deep Reason

Experimental Results

Explainable AI

Promising Topics

Smart Precision Imaging/Medicine



FIGURE 2. Big picture of deep imaging – A full fusion of medical imaging and deep learning. A high likelihood is that the direct paths from data to features and actions may need an intermediate layer essentially equivalent to a reconstructed/processed image.

Rawdiomics



Kalra M, Wang G: Radiomics in Lung Cancer: Its Time Is Here. Med. Phys., DOI: 10.1002/mp.12685, 2017 In Collaboration with Amber Simpson (MSK), Bruno De Man (GE GRC), Pingkun Yan (RPI), Mannudeep Kalra (MGH), et al.

End-to-end CT Imaging

END-TO-END ABNORMALITY DETECTION IN MEDICAL IMAGING

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Figure 4: The schematics for joined training of the reconstruction and detection neural networks. Red arrows stand for backpropagation through neural networks. $g_{\theta_{r1}}, ..., g_{\theta_{rK}}$ stand for the gradients accumulated in step 16 in algorithm 1.

bicmr.pku.edu.cn/~dongbin/Publications/EndToEndMedical.pdf

Direct Sinogram Analysis

Analysis of Blood Vessel Features in the CT Sinogram via Deep Learning

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Figure 8: Example sinograms from the testing phase of the simulation study: (a) An input sinogram, (b) the label sinogram, (c) estimated sinogram and (d) error sinogram. All sinograms are displayed in a [0 - 1.0] interval.

Deep Sinogram Analysis



Energy-integrating vs Photon-counting CT


Photon-counting Spectral CT

NIH HEI 1S100D026811Wang G (PI)04/01/19 – 03/31/20Acquisition of MARS Photon-counting Micro-CT ScannerThe goal is to acquire the state of the art MARS photon-counting micro-CT scanner to support
major users who work on NIH-funded R01s and other research projects.



A Clinical Trial of 400 Patients in NZ (RPI plans to receive data)

Emulation on CT Benchtop (Donated by GE-GRC)





X-ray Source 60kW GE tube for 64-slice CT High-V generator, 40-140kVp (JEDI) Detector/DAS Partial GE VCT-LightSpeed detector 64-slice (Z) by 128 pixels (X), 1x1 mm² **Motion Stages** Source: X, Y, Z, and φ Detector: X, Y, and φ Phantom: Z and φ





Left Coronary artery



http://www.cirsinc.com

Simulated Example



Meng B, Yang J, Ai DN, Fu TY, Wang G Beijing Institute of Technology, Beijing, China; Rensselaer Polytechnic Institute, Troy, NY, USA



Na YH, Zhang B, Zhang J, Caracappa PF, Xu XG: Deformable Adult Human Phantoms for Radiation Protection Dosimetry: Anatomical Data for Covering 5th- 95th Percentiles of the Population and Software Algorithms. Phys. Med. Biol. 55: 3789-3811, 2010

Limerick: Lady of Niger



There was a young lady of Niger Who smiled as she rode on a tiger; They returned from the ride With the lady inside, And the smile on the face of the tiger.

Natural Language Processing (NLP)



Semantic Tomography

Machine Learning for Tomographic Imaging Ge Wang, Yi Zhang, Xiaojing Ye, & Xuanqin Mou



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National Institutes of Health







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