Sonic reflection imaging in the time domain

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The model problem in sonic imaging: Inverse scattering

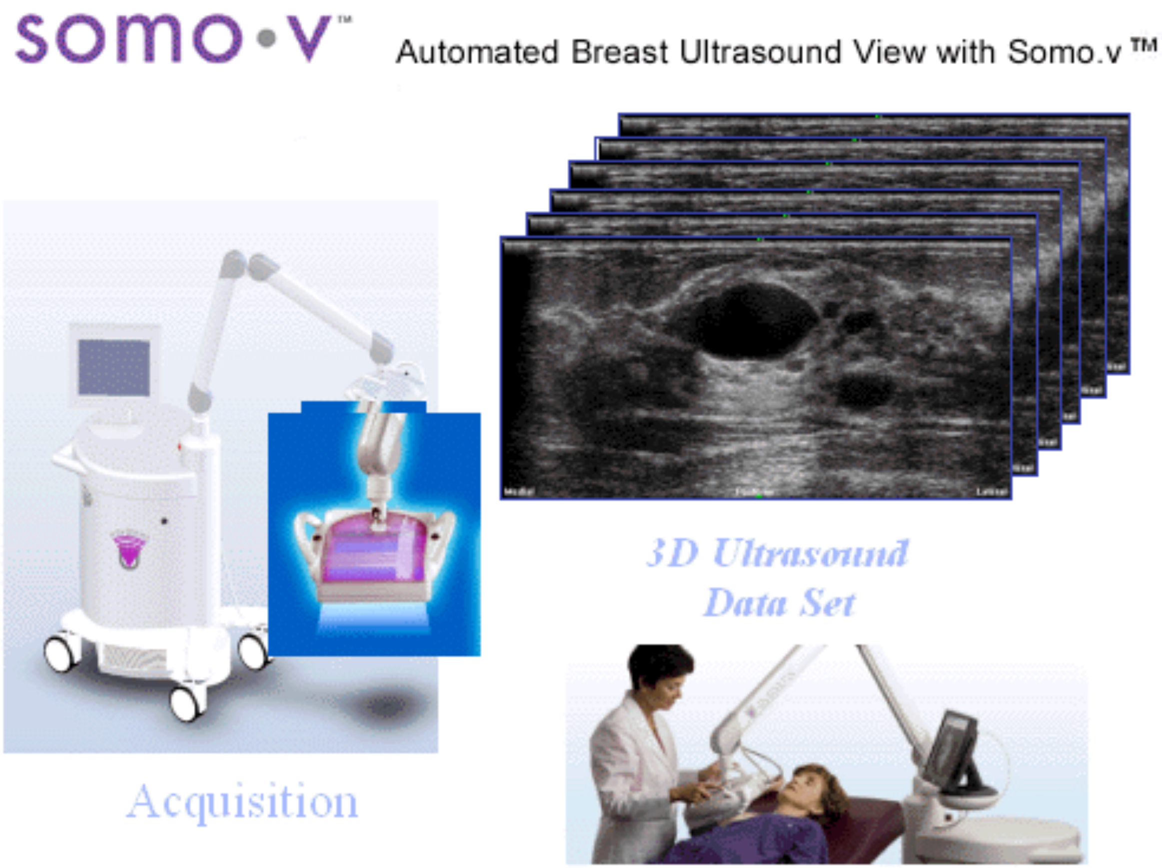
$$\frac{\partial^2 u}{\partial t^2}(x,t) = c^2(x)(\Delta u(x,t) + \delta(x-s)q(t))$$

$$g_s(x',t) = u(x',D,t) = R_s(f)(x',t)$$

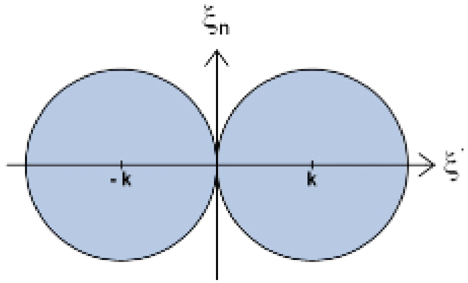
$$c(x) = c_0/(1+f(x))$$

Mammography reflection imaging

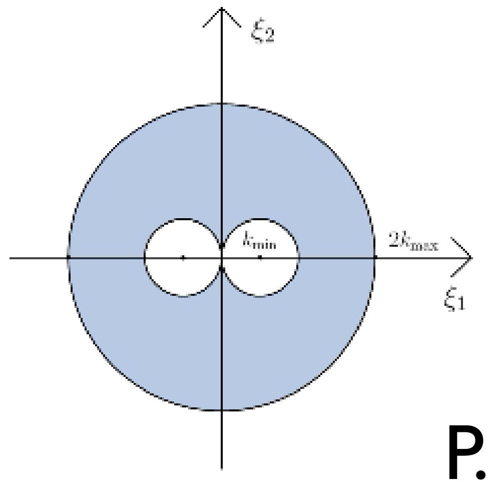
3D scanner of U-Systems



Coverage in Fourier domain



Transmission



Reflection

P. Mora , 1989

$$\hat{f}(\sigma+\rho, a(\sigma)-a(\rho)) \qquad \hat{f}(\sigma+\rho, a(\sigma)+a(\rho))$$

$$a(\sigma)=\sqrt{k^2-\sigma^2}$$

What can we do about missing low frequencies?

10 kHz - 150 kHz

Kaczmarz' method for nonlinear problems (consecutive time reversal)

Solve $R_s(f) = g_s$ for all sources s.

Update:

$$f \longleftarrow f - \alpha (R_s'(f))^* (R_s(f) - g_s)$$

$$\frac{\partial^2 z}{\partial t^2} = c^2(x)\Delta z \text{ for } x_2 > 0,$$

$$\frac{\partial z}{\partial x_2} = r \text{ on } x_2 = 0,$$

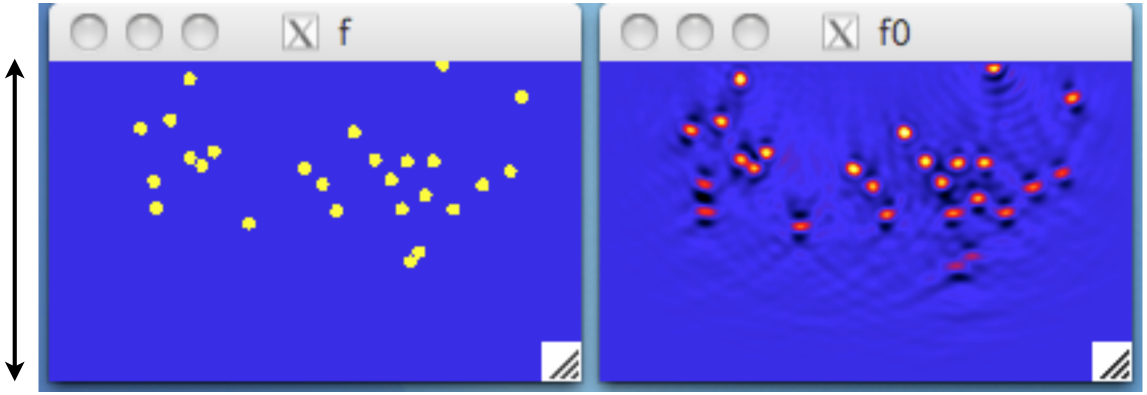
$$z = 0 \text{ for } t > T.$$

Compute the adjoint by time reversal:

$$(R_s'(f))^* r)(x) = \int_0^T z(x,t) \frac{\partial^2 u(x,t)}{\partial t^2} dt$$

Easy case Nr. 1: Clutter

Original
12 cm



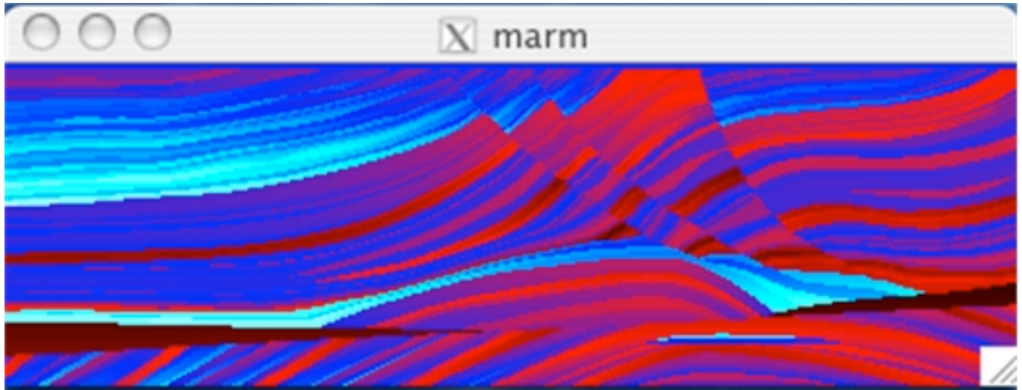
5 sweeps of
Kaczmarz

Diameter of
dots 5 mm

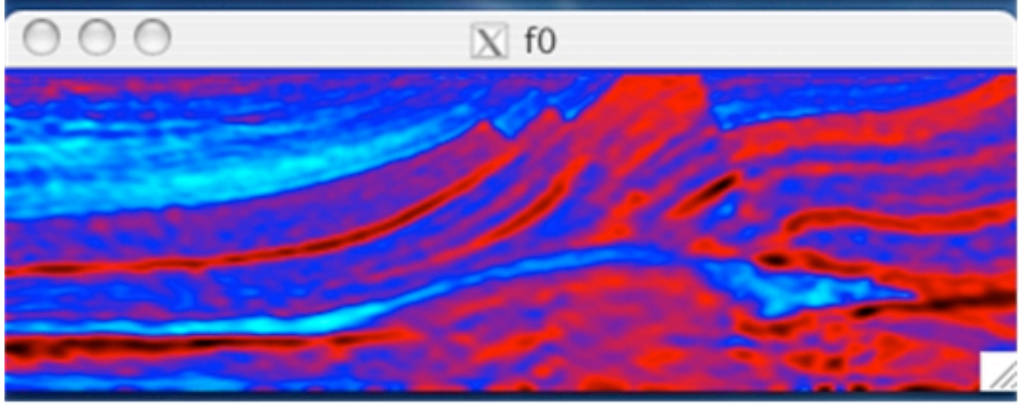
Frequency range 50 to 150 kHz
ambient speed of sound 1500m/s
wave length 1cm

Easy case Nr. 2: Source wavelet q is Gaussian peak.

Original



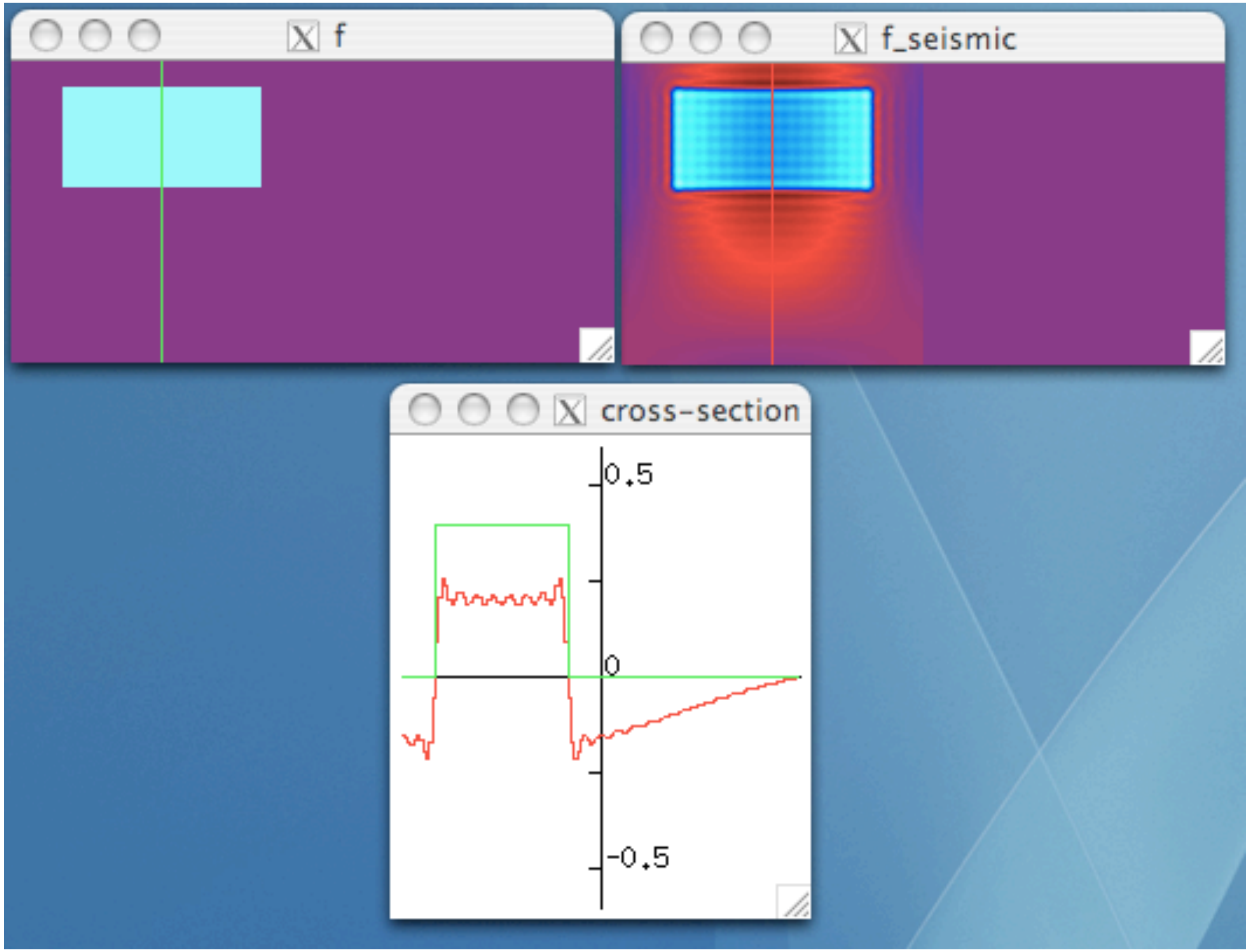
6 sweeps



original reconstruction

Difficult case

10 kHz - 150 kHz



What can reflections do in mammography?

aperture 20cm

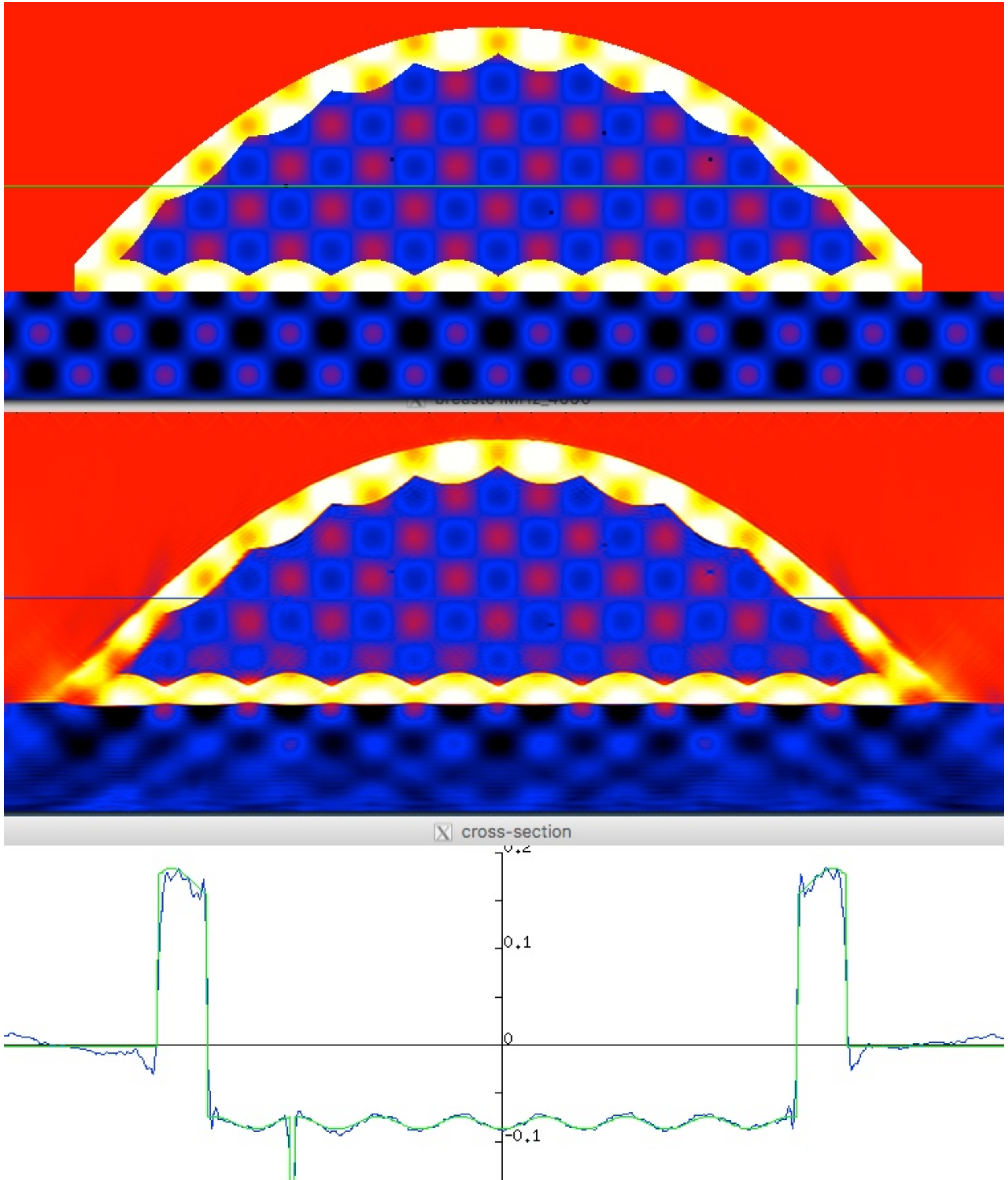
depth 5cm

frequency range 15kHz - 1MHz

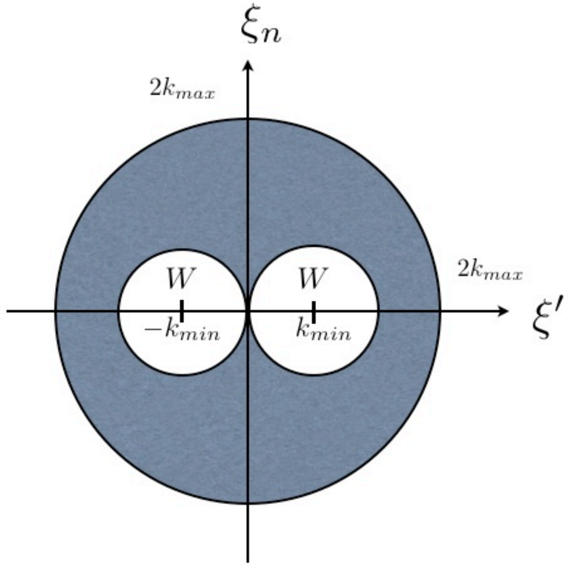
wavelength 1.5 mm

tumor 0.75 mm

stepsize 0.25 mm



Idea 1: Fill the white circles W by analytic continuation!



$$
\begin{aligned}
f(x) &= (2\pi)^{-n/2}\int_W e^{ix\cdot\xi}\hat{f}(\xi)d\xi + (2\pi)^{-n/2}\int_{R^n\setminus W} e^{ix\cdot\xi}\hat{f}(\xi)d\xi \\
&= \int_{|x|<r} K(x-y)f(y)dy + g(x), \\
K(x) &= 2(2\pi)^{-n/2}cos(kx_1)\frac{J_{n/2}(k|x|)}{(k|x|)^{n|2}}
\end{aligned}
$$

Gerchberg-Papoulis

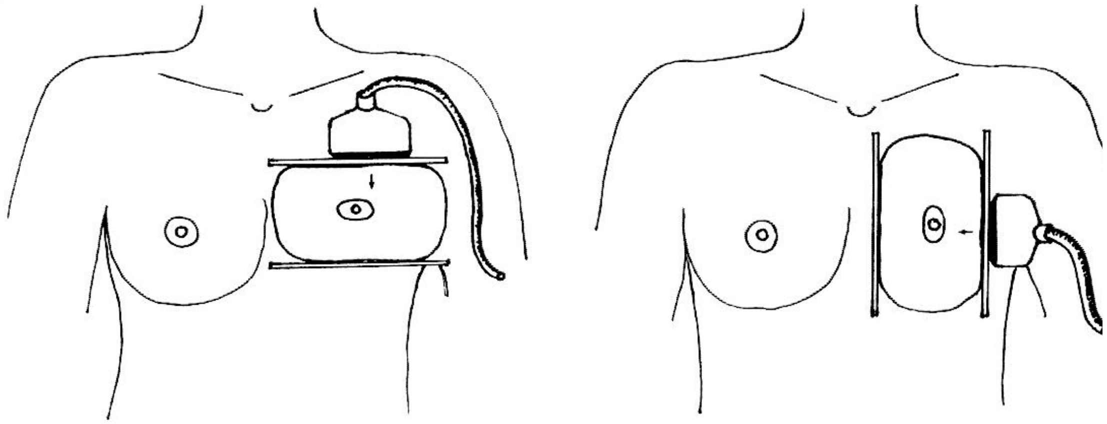
Idea 2: Use reflector

CARI (K. Richter, 1996)
Clinical amplitude/velocity reconstructive imaging
H. Madjar (2018)
Challenges in Breast Ultrasound

Ovid®

Wolters Kluwer

Support & Training Feedback Close



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The model problem with a reflector

$$\frac{\partial^2 u}{\partial t^2}(x,t) = c^2(x)(\Delta u(x,t) + \delta(x-s)q(t))$$

$$g_s(x',t) = u(x',D,t) = R_s(f)(x',t)$$

$$\frac{\partial u}{\partial x_n}(x',0,t) = 0$$

Born approximation

$G(x,y)$ free space Green's function

$G_0(x,y)$ Green's function for reflector, $x=(x',x_n), y=(y',y_n)$:

$$G_0(x,y)=G(x'-y',x_n-y_n)+G(x'-y',x_n+y_n)$$

$$\tilde{u}(x)-\tilde{u}_0(x)=k^2\tilde{q}(\omega)\int_{0<y_n<D}G_0(x,y)f(y)\tilde{u}_0(y)dy, k=\omega/c_0$$

Plane wave decomposition of Green's function

$$\hat{G}(\xi',x_n)=i(2\pi)^{(n-1)/2}c_n\frac{e^{i|x_n|a(\xi')}}{a(\xi')},$$

$$a(\xi')=\sqrt{k^2-|\xi'|^2}, c_2=1/(4\pi), c_3=1/(8\pi^2)$$

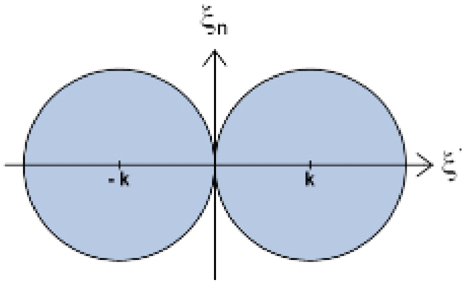
Resulting integral equation:

$$data(\rho,\sigma) = (C\hat{f})(\rho+\sigma, a(\rho)+a(\sigma)) + (C\hat{f})(\rho+\sigma, a(\rho)-a(\sigma))$$

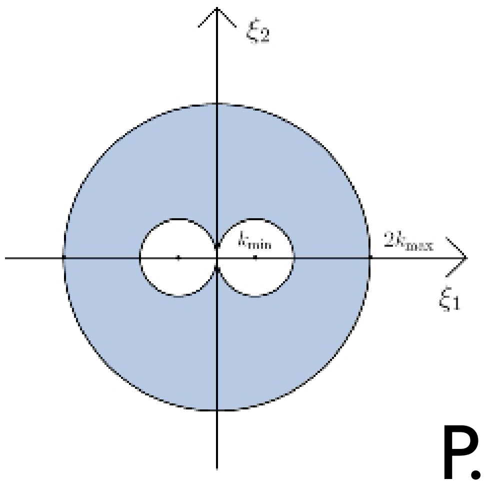
$\hat{f}$ n -dimensional Fourier-transform of $f, |\rho|, |\sigma| \leq k$

C Cosine-transform with respect to last argument

Coverage in Fourier domain



Transmission



Reflection

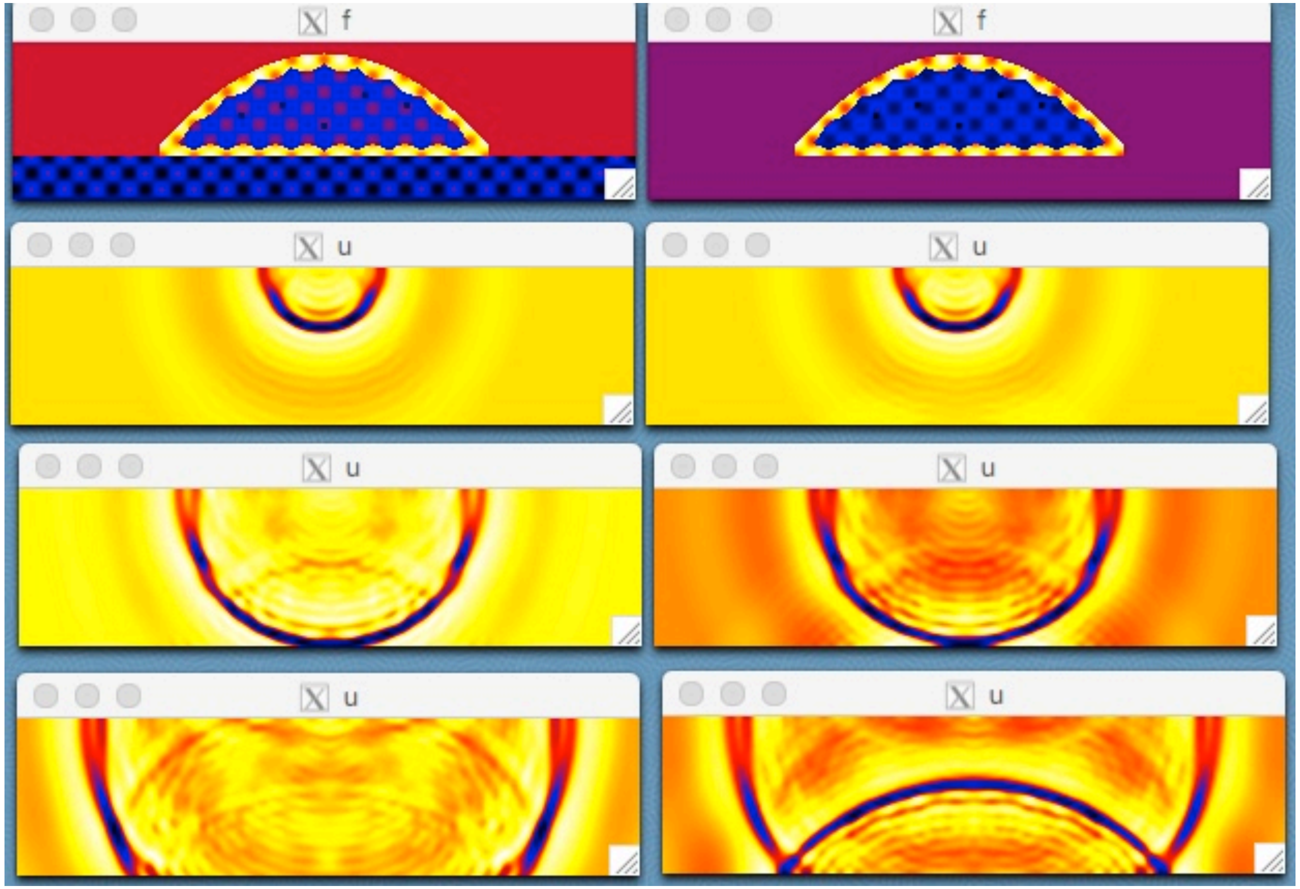
P. Mora , 1989

$$\hat{f}(\sigma + \rho, a(\sigma) - a(\rho)) \qquad \hat{f}(\sigma + \rho, a(\sigma) + a(\rho))$$

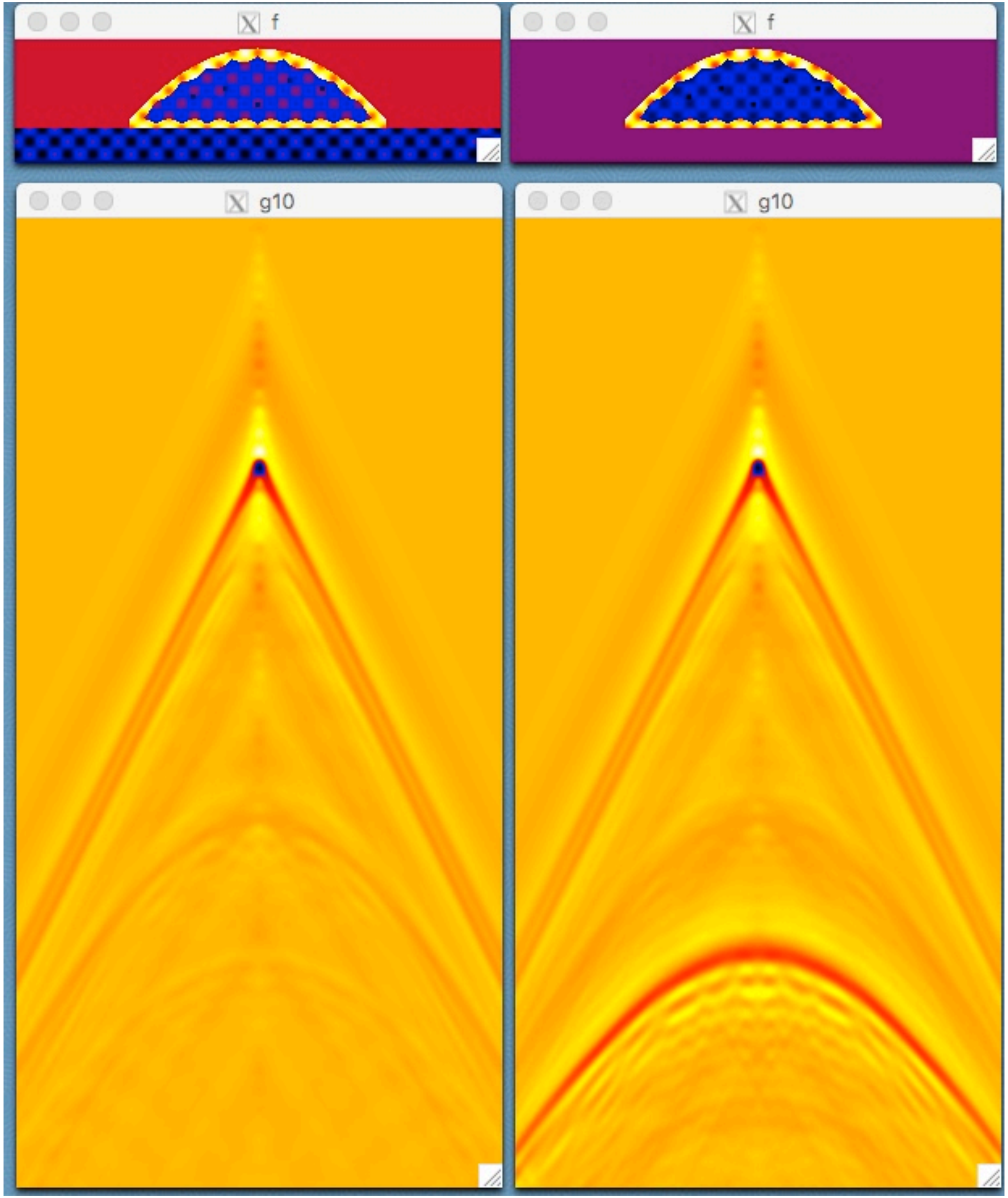
$$a(\sigma) = \sqrt{k^2 - \sigma^2}$$

Waves from 1 source on the top boundary

No reflector **Reflector**



Data without reflector



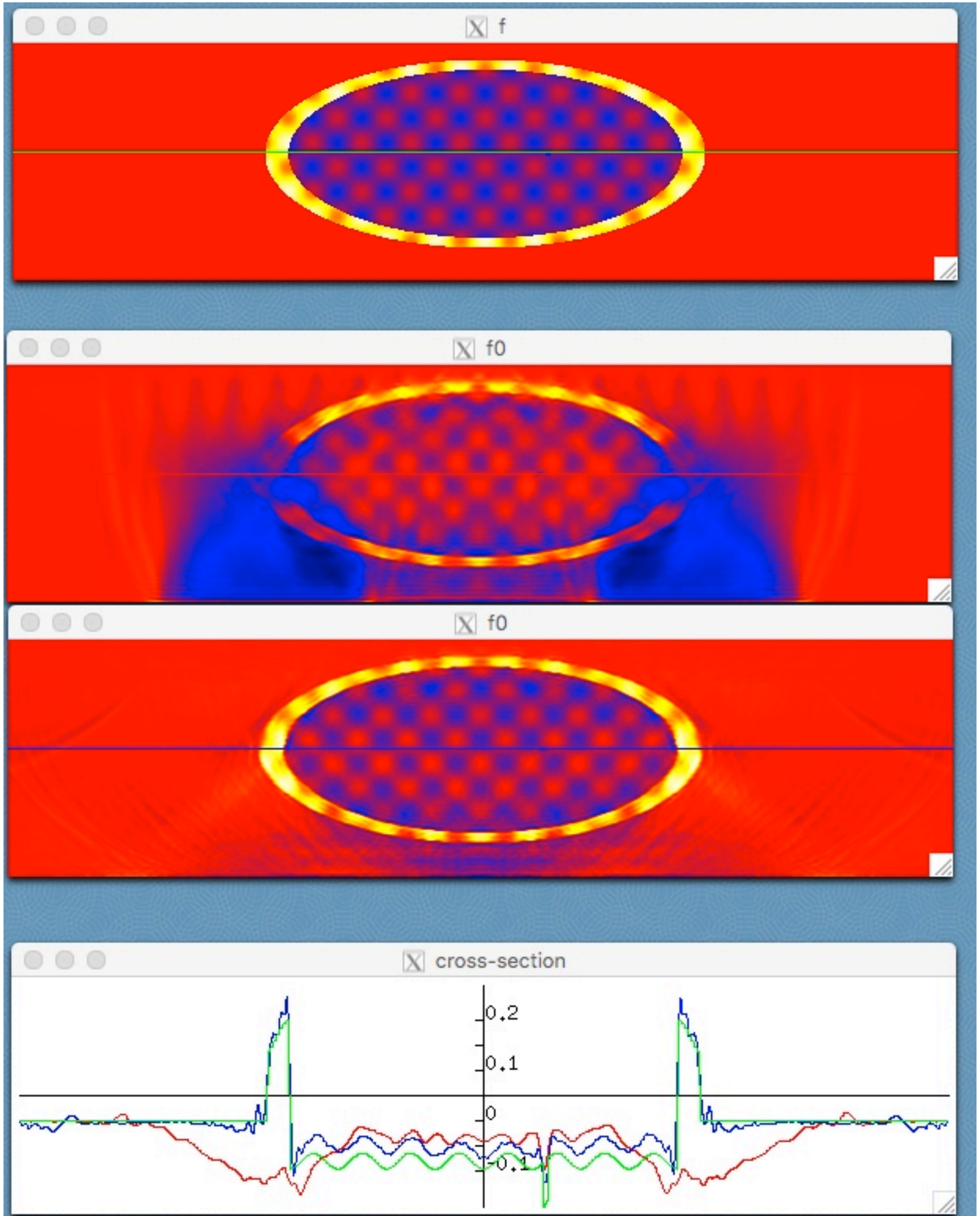
Data with reflector

original
tumors 1.5 mm

reconstruction
without reflector
30-500 kHz

reconstruction
with reflector
30-500 kHz

cross sections



Layered medium

$f(x_1, x_2) = f(x_2)$.

Born approximation, one source at $x_1 = 0,\ x_2 = 0$:

$g_k(x) = (2\pi)^{-1/2} \int e^{-ix\xi} \hat{f}(-2\kappa(\xi))d\xi,\ \kappa = \sqrt{k^2 - \xi^2}$.

Finite aperture: Data available for $|x| \leq A$ only.

All we can determine: $\int \delta_A(\eta - \xi)\hat{f}(-2\kappa(\xi)d\xi,\ \delta_A(\xi) = \frac{A}{\pi} sinc(A\xi)$.

Determine $\hat{f}$ from

$$\int \delta_A(\eta-\xi)\hat{f}(-2\kappa(\xi))d\xi,\ \delta_A(\xi)=\frac{A}{\pi}sinc(A\xi),\ \kappa=\sqrt{k^2-\xi^2}.$$

peaks in η, bandwidth A

$\hat{f}(-2\kappa(\xi))$ can be stably

determined for $A > 2z|\xi|/\kappa(\xi)$

i.e. $\frac{2k}{\sqrt{1+A^2/4z^2}} < 2\kappa < 2k.$

for line object at depth z:

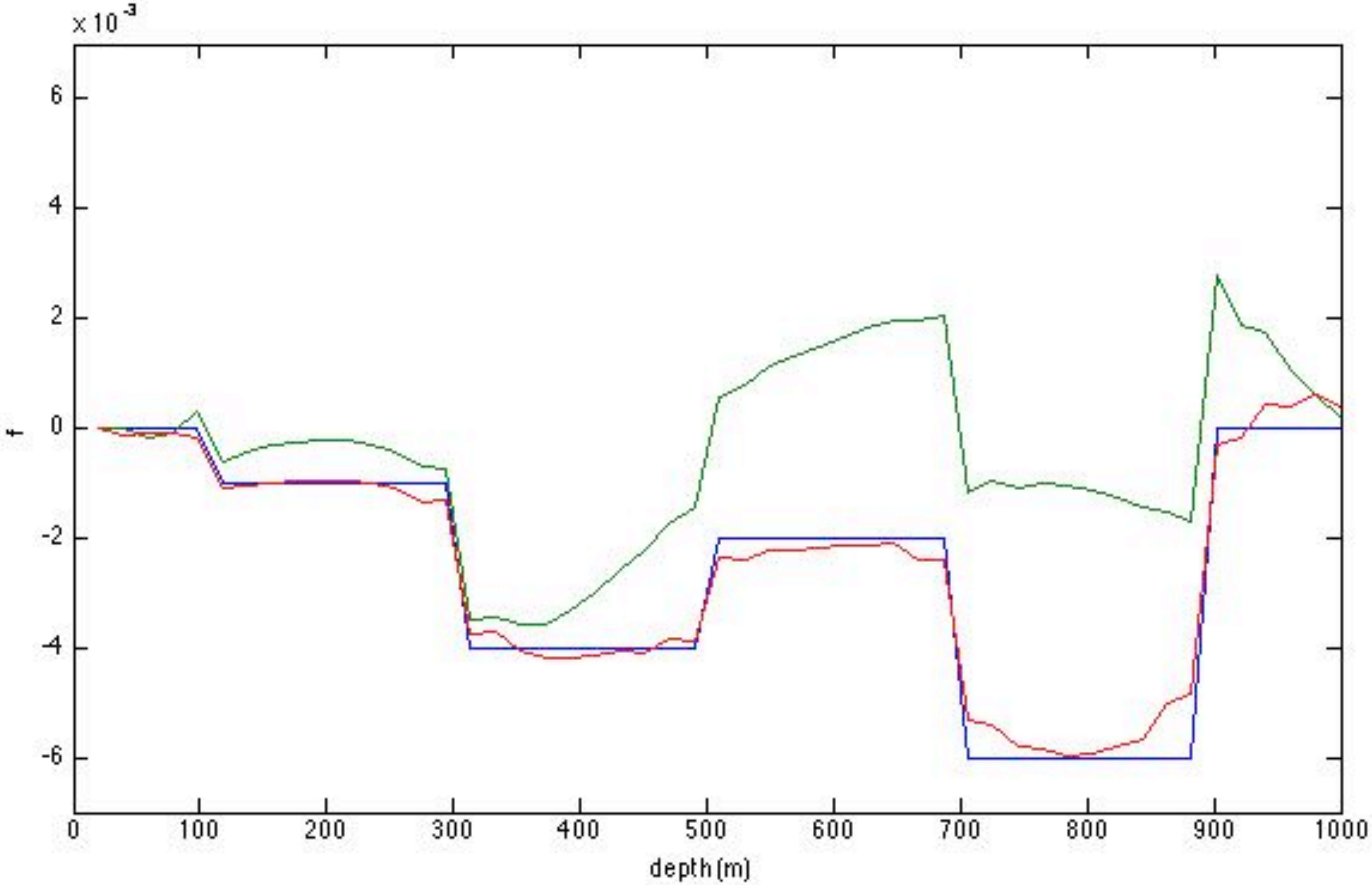
$f(x)=\delta(x-z),\ \hat{f}(\xi)\sim e^{-iz\xi},$

$\hat{f}(-2\kappa(\xi))\sim e^{-2iz\kappa(\xi)}$ for $|\xi|<k.$

bandwidth $2z|\kappa'(\xi)| = 2z|\xi|/\kappa(\xi)$

Sirgue & Pratt 2004

Kaczmarz‘ method, frequencies 5-25 Hz



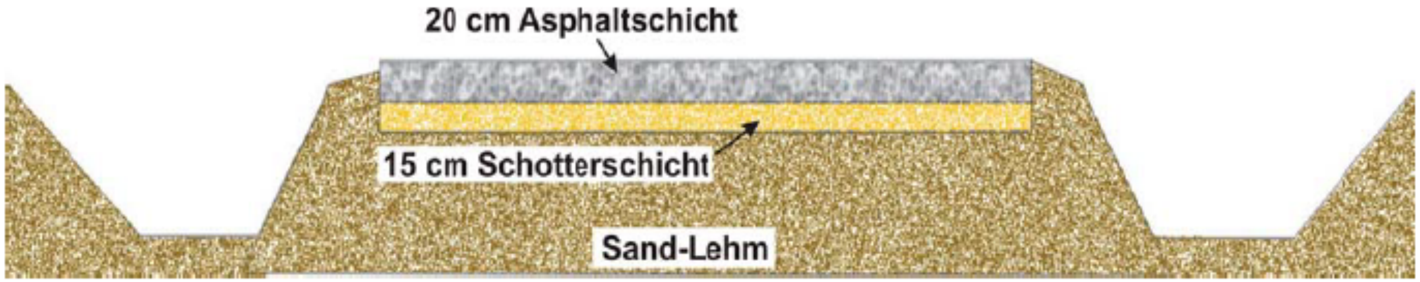
true profile

Kaczmarz starting at f=0

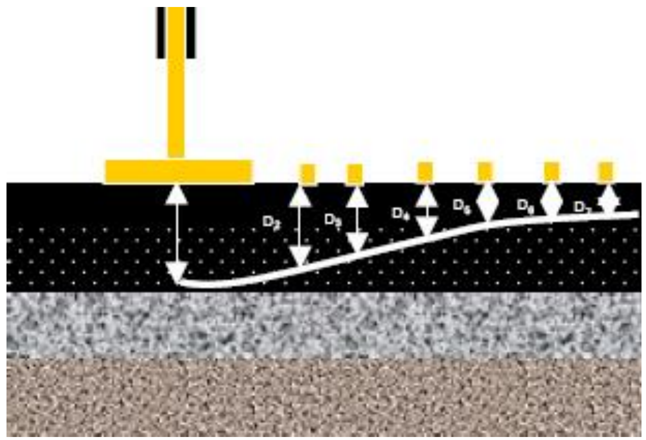
Kaczmarz with analytic continuation

aperture 12 km, wave length 400 m

BASt



Falling weight deflectometer (FWD)



Conclusion:

Reflection imaging without low frequencies can be improved by

reflectors
big apertures
analytic continuation