Star Radon Transform

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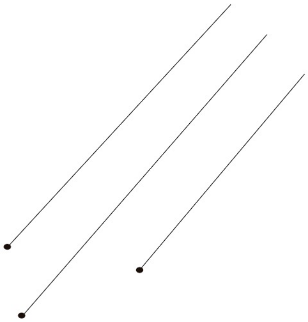
Joint work with Gaik Ambartsoumian.

Previous Works

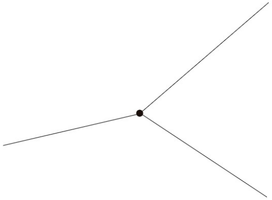
1. Zhao F, Schotland J C and Markel V A 2014 Inversion of the star transform Inverse Problems 30 105001
2. The V-line transform with some generalizations and cone differentiation, Gaik Ambartsoumian, MJ Latifi, Inverse Problems, Vol.35 (3),2019

- **Radon Transform** $\mathcal{R}$: Integrate over lines
- **Ray Transform** $\mathcal{X}_\gamma$: Integrate over half-lines with fixed direction



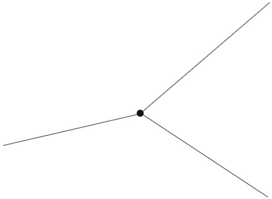
Star Radon Transform

Star Radon Transform $\mathcal{S}$: Integrate over stars, i.e. sum of integrals of the function along a collection of rays emanating from the given input point.



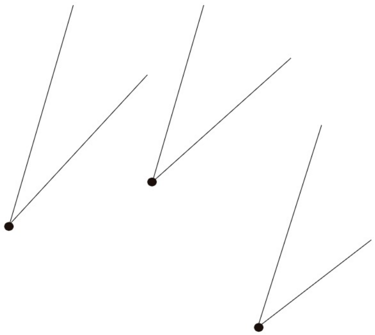
Weighted Star Radon Transform : Weighted sum of Ray transforms

Star Radon Transform $\mathcal{S}$: Integrate over stars, i.e. sum of integrals of the function along a collection of rays emanating from the given input point.



Weighted Star Radon Transform : Weighted sum of Ray transforms

V-line Radon Transform : When we have only two branches.



Imaging Setup

Star Radon Transform first proposed by Zhao, Schotland and Markel in [1]. Applied in imaging setups involving scattering.

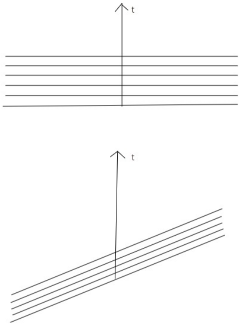
The Inversion Technique

Basic Ingredients :

- Moving Section Lemma ([2] Ambartsoumian and Latifi 2019)
- Geometric arguments involving integration and differentiation and standard Radon transform.

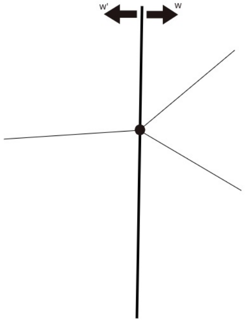
Moving Section Lemma

- $\int g(t)\,dt = c$

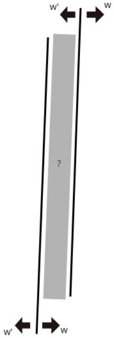


- $\int g(t)\,dt = \frac{1}{\sin(\theta)} c$
- Rigorous measure theoretic proof in [2]

Step 1. Apply Standard Radon Transform.
When we integrate the Star Radon Transform Sf along a line, we obtain weighted integral of f on each side of the line.



Step 2. Find the Radon Transform of f, possibly with some weight factor depending on angle.



Theorem

Let $\mathcal{S} = \sum_{i=1}^{m} \mathcal{X}_{\gamma_i}$ *be the star transform and let*

$$q(\psi) = \frac{-1}{\sum_{i=1}^{m} \frac{1}{\langle \psi, \gamma_i \rangle}}$$

Then the following is true for any ψ *in the domain of* q

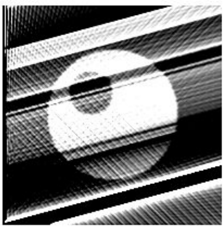
$$\mathcal{R}f(\psi, s) = q(\psi) \frac{d}{ds} \mathcal{R}(\mathcal{S}f)(\psi, s)$$

and hence if q *is defined almost everywhere we can apply* $\mathcal{R}^{-1}$ *to get a reconstruction.*

Original Phantom (300x300)

Reconstruction with branch angles : $0°,\ 135°,\ 270°$

Reconstruction with branch angles : $0^\circ,\ 30^\circ$



Reconstruction with branch angles : $0^\circ,\ 30^\circ,\ 135^\circ,\ 270^\circ$

Thanks

Thank you for your attention !