Magnetoacoustic and magnetoelectric hybrid imaging modalities

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Hybrid magnetic modalities: motivation

Imaging of electric conductivity is of high interest for cancer diagnostics.

Conductivity in tumors is much higher than that in healthy tissues
$\Longrightarrow$ conductivity imaging can yield high contrast.

Electrical impedance tomography (EIT) and other electromagnetic only modalities lead to strongly non-linear and ll-posed inverse problems

Idea: Use hybrid techniques, couple electromagnetism with either ultra-sound or magnetic resonance (MRI).

Hybrid magnetic modalities: a short list

Below the better known modalities are listed first:

MRI based hybrids:

Magnetic resonance electrical impedance tomography (MREIT)
Current density impedance imaging (LF-CDII and RF-CDII)
Magnetic resonance electrical properties tomography (MREPT)

Ultrasound hybrids:

Magnetoacoustic Tomography with Magnetic Induction (MAT-MI)
Magnetoacoustoelectric Tomography (MAET aka LFEI)
Magnetoacoustic Tomography with Current Injection (MAT-CI)
Magnetoacoustoelectric Tomography with Coil Pickup (MAET-MI)

Nice review:

"Hybrid tomography for conductivity imaging"
T. Widlak and O. Scherzer, *Inverse Problems* **28** (2012) 08400

Currents and fields in MRI-based modalities tissues

The object is placed in an MRI scanner.

Current $\mathbf{J}(t, \mathbf{x})$ is generated in the object either by applying a potential on the boundary, or (recently) by a coil exciting eddy currents ...

There are no sinks or sources of charges inside the object, so

$$\nabla \cdot \mathbf{J}(t, \mathbf{x}) = 0.$$

The current excites a (weak) magnetic field $\mathbf{B}_1(t, \mathbf{x})$ (under quasistatic approximation)

$$\nabla \times \mathbf{B}_1(t, \mathbf{x}) = \mu_0 \mathbf{J}(t, \mathbf{x}),$$

where μ_0 is magnetic permitivity (assumed constant in $\mathbb{R}^3$).

Or (by Biot-Savart law),

$$\mathbf{B}_1(t, \mathbf{x}) = \frac{\mu_0}{4\pi} \int \mathbf{J}(\mathbf{y}, t) \times \frac{\mathbf{x} - \mathbf{y}}{|\mathbf{x} - \mathbf{y}|^3} d\mathbf{y}, \qquad \mathbf{x} \in \mathbb{R}^3.$$

Data acquisition in MREIT, MREPT, and CDI

In the scanner, there isa strong constant magnetic field $\mathbf{B}_0(\mathbf{x}) = (0, 0, B_0(\mathbf{x}))$. produced by the main coil of the machine, with $\mathbf{B}_0 \gg \mathbf{B}_1$

The normal MRI scan is conducted. Then it is repeated with $\mathbf{J}(\mathbf{x})$ replaced by $-\mathbf{J}(\mathbf{x})$.

Without the current, the MRI machine would produce a structural image of the object. The induced magnetic field $\mathbf{B}_1$ distorts the image. By subtracting the measurements obtained with $\mathbf{J}(\mathbf{x})$ and $-\mathbf{J}(\mathbf{x})$ one is able to recover spatial distribution of $\vec{e}_3 \cdot \mathbf{B}_1$ (a vertical component of $\mathbf{B}_1$).

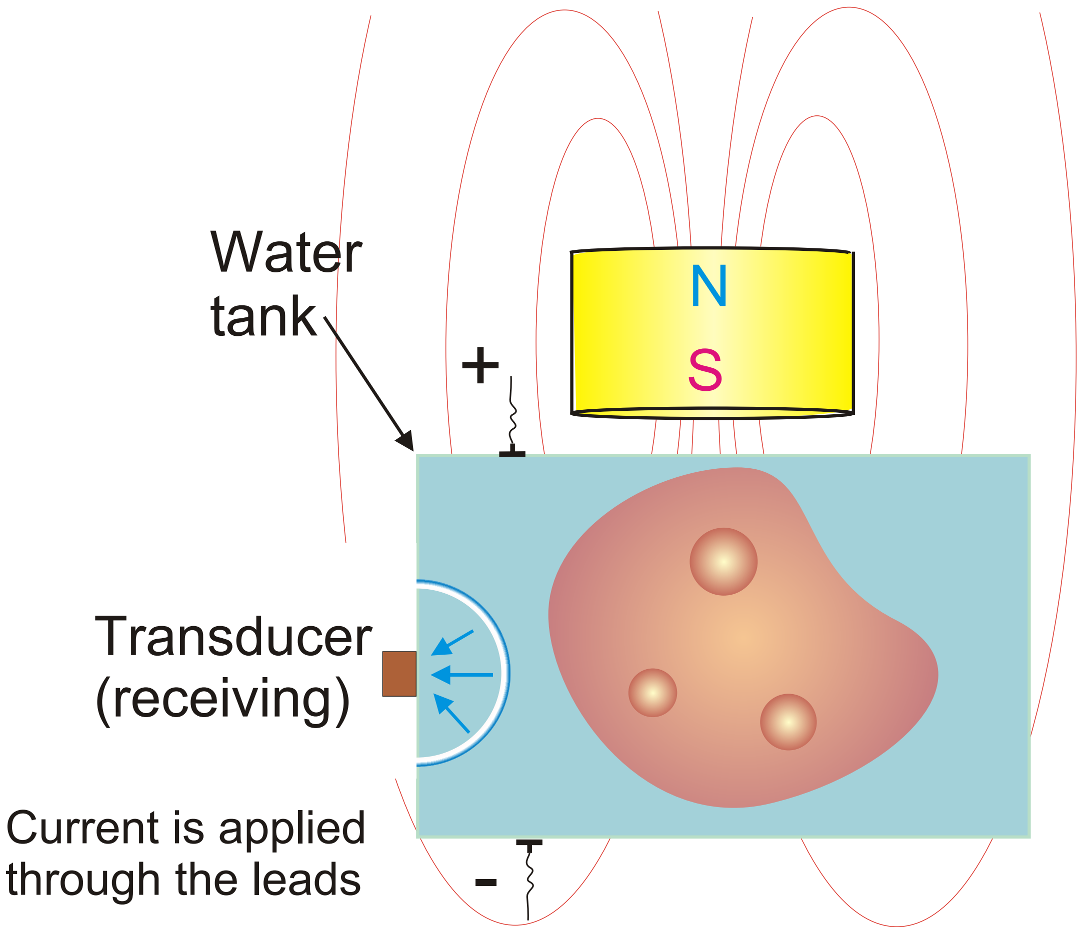
By rotating the object in the MRI machine, one can get the whole $\mathbf{B}_1$ and hence $\mathbf{J}(\mathbf{x})$ (CDI). But it would be nice to avoid the rotation!!!

Alternatively, one repeats the measurements with different currents $\mathbf{J}^{(k)}(\mathbf{x})$, $k = 1, 2, ...$ to obtain $\vec{e}_3 \cdot \mathbf{B}_1^{(k)}$, $k = 1, 2, ...$

Acoutic coupling: MAT-CI

(Magnetoacoustic Tomography with Current Injection)

MRI machine is too expensive!
Instead, generate an acoustic wave in the object and record the pressure.



The source of the acoustic wave is the Lorentz force.

MAT-CI (continued)

Current = free charges moving in a magnetic field $\mathbf{B}_0$.

Lorentz force for a single charge q: $\mathbf{F} = q\mathbf{V} \times \mathbf{B}_0$.

Lorentz force for a current $\mathbf{J}(t, \mathbf{x})$: $\mathbf{F}(t, \mathbf{x}) = \mathbf{J}(t, \mathbf{x}) \times \mathbf{B}_0$.

For the pressure $p(t, \mathbf{x})$ the Euler equation + mass conservation yield

$$\Delta p(t, \mathbf{x}) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} p(t, \mathbf{x}) = \nabla \cdot \mathbf{F}(t, \mathbf{x}) = \nabla \cdot [\mathbf{J}(t, \mathbf{x}) \times \mathbf{B}_0] = \mathbf{B}_0 \cdot \nabla \times \mathbf{J}(t, \mathbf{x}).$$

I.e. the source term is a projection on $\mathbf{B}_0$ of the curl $\nabla \times \mathbf{J}(t, \mathbf{x})$.

If the current is pulsed, i.e. $\mathbf{J}(t, \mathbf{x}) = \delta'(t)\mathbf{J}(\mathbf{x})$, Cauchy problem:

$$\Delta p(t, \mathbf{x}) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} p(t, \mathbf{x}) = 0, \qquad p(0, \mathbf{x}) = \mathbf{B}_0 \cdot \nabla \times \mathbf{J}(\mathbf{x}), \qquad p_t(0, \mathbf{x}) = 0,$$

where $p(t, \mathbf{z})$ is measured at all $\mathbf{z} \in \Sigma$ (a measurement surface)

MAT-CI (continued)

Reconstructing $\mathbf{B}_0 \cdot \nabla \times \mathbf{J}(\mathbf{x})$ from $p(t, \mathbf{z})$, $\mathbf{z} \in \Sigma$ is the inverse source problem similar to TAT/PAT (thermo- and photo- acoustic tomography)!!!

In order to obtain enough data the transducer should move along Σ.

Good news:
one can simultaneously use several transducers or a transducer array.

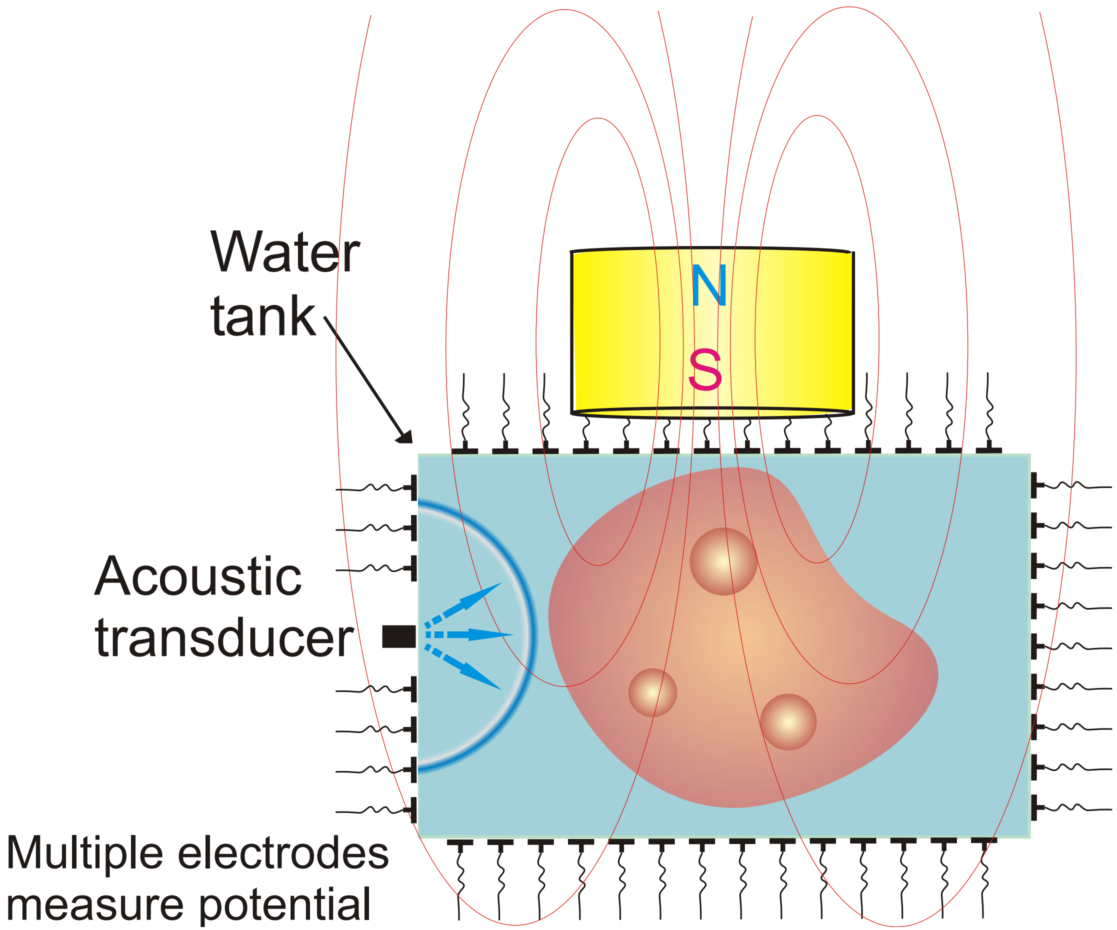
In order to obtain curl $\nabla \times \mathbf{J}(\mathbf{x})$ one has to repeat measurements at least two times with a rotated object or rotated $\mathbf{B}_0$.

In order to reconstruct several curls $\nabla \times \mathbf{J}^{(k)}(\mathbf{x})$, $k = 1, ..3$ one has to repeat measurements with different injected currents $\mathbf{J}^{(k)}(\mathbf{x})$.

MAET (Magnetoacoustoelectric Tomography)

MAET = MAT-CI in reverse.
The transducer is transmitting ultrasound pulses.
The Lorentz force generates an electric current.

The resulting electric potential is measured on the boundary.



Physics & mathematics of MAET

Charges vibrating with velocity $V(t, \mathbf{x})$ produce Lorentz currents $\mathbf{J}_L(t, \mathbf{x})$:

$$\mathbf{J}_L(t, \mathbf{x}) = \sigma(\mathbf{x})\mathbf{V}(t, \mathbf{x}) \times \mathbf{B}_0$$

The total current $\mathbf{J}$ is a sum of $\mathbf{J}_L$ and the Ohmic current $\mathbf{J}_O = \sigma(\mathbf{x})\nabla u(t, \mathbf{x})$:

$$\mathbf{J} = \mathbf{J}_O + \mathbf{J}_L,$$

where $u(t, \mathbf{x})$ is the electric potential.

Since $\nabla \cdot \mathbf{J} = 0$,

$$\nabla \cdot \sigma(\mathbf{x})\nabla u(t, \mathbf{x}) = -\nabla \cdot (\sigma(\mathbf{x})\mathbf{V}(t, \mathbf{x}) \times \mathbf{B}_0) \,.$$

BC: there is no current throught the boundary, $n \cdot \mathbf{J}(t, \mathbf{z}) = 0$ for $\mathbf{z} \in \partial\Omega$.

Potential $u(t, \mathbf{x})$ is measured at several points on the boundary.
However, only difference of potentials makes a physical sense.

Reciprocity

Suppose we are measuring the difference of potentials $M(t) \equiv u(\mathbf{z}_2, t) - u(\mathbf{z}_1, t)$, for $\mathbf{z}_1, \mathbf{z}_2 \in \partial\Omega$.

Consider a virtual current $\mathbf{J}_{\text{Lead}}(\mathbf{x})$ that would flow in the object if a unit current were injected at the point $\mathbf{z}_2$ and extracted at the point $\mathbf{z}_1$.

Using the second Green's identity (= **reciprocity** principle) one can show

$$M(t) \equiv u(\mathbf{z}_2, t) - u(\mathbf{z}_1, t) = \int_{\Omega} \mathbf{B}_0 \cdot \mathbf{J}_{\text{Lead}}(\mathbf{x}) \times \mathbf{V}(t, \mathbf{x}) d\mathbf{x}.$$

In MAET the measurement tests a virtual current $\mathbf{J}_{\text{Lead}}(\mathbf{x})$ as opposed to real currents in MAT-CI.

More on MAET

Assume that speed of sound c and density ρ are constant.

Then, velocity is the gradient of the velocity potential $\varphi(t, \mathbf{x})$:

$$\mathbf{V}(t, \mathbf{x}) = \frac{1}{\rho}\nabla\varphi(t, \mathbf{x}),$$

where velocity potential $\varphi(t, \mathbf{x})$ is the time anti-derivative of pressure $p(t, \mathbf{x})$:

$$p(t, \mathbf{x}) = \frac{\partial}{\partial t}\varphi(t, \mathbf{x}).$$

This leads to

$$M(t) \equiv u(\mathbf{z}_2, t) - u(\mathbf{z}_1, t) = \frac{1}{\rho}\mathbf{B}_0 \cdot \int\limits_{\Omega} \varphi(t, \mathbf{x})\nabla \times \mathbf{J}_{\text{Lead}}(\mathbf{x})d\mathbf{x}$$

Since $\varphi(t, \mathbf{x})$ solves the wave equation, the problem of finding $\mathbf{B}_0 \cdot (\nabla \times \mathbf{J}_{\text{Lead}}(\mathbf{x}))$ is the same inverse source problem as before.

Mathematically, MAT-CI and MAET are identical. However...

MAET vs MAT-CI

In MAET, one can measure several potential differences simultaneously, and obtain the data to recover $\mathbf{B}_0 \cdot (\nabla \times \mathbf{J}^{(k)}_{\text{Lead}}(\mathbf{x}))$, $k = 1, 2, 3, ..$

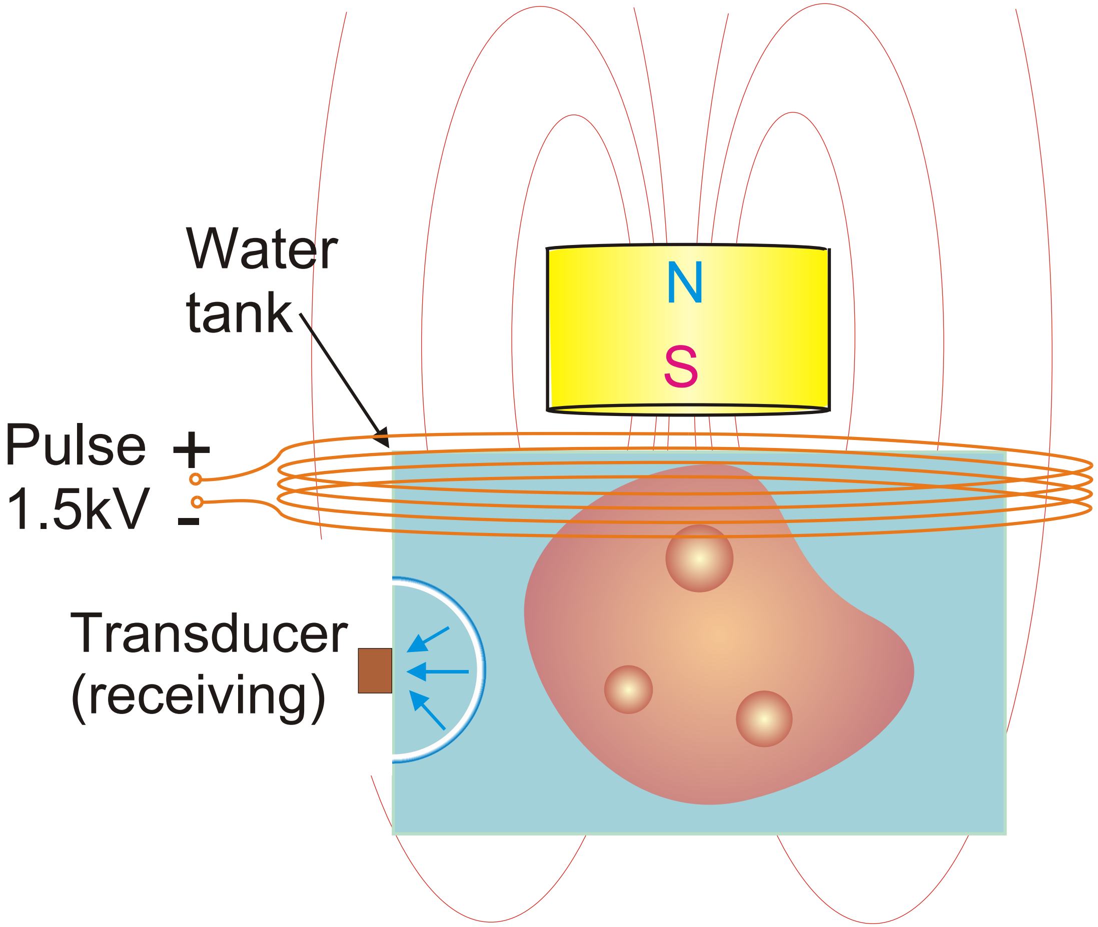
However, only one acoustic field can be utilized at a time.

In MAT-CI one can use several receiving transducers, but only one current $\mathbf{J}^{(k)}(\mathbf{x})$ at a time.

A significant practical problem in MAT-CI is the need for a relatively strong current = safety issue. Very few experimental works exist at the time...

MAT-MI (Magnetoacoustic tomography with magnetic induction)

MAT-MI is like MAT-CI but: ... instead of injecting a current, an eddy current is induced by a strong magnetic pulse.



More on MAT-MI

Similarly to MAT-CI, the acoustic pressure $p(t, \mathbf{x})$ solves the Cauchy problem:

$$\Delta p(t, \mathbf{x}) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} p(t, \mathbf{x}) = 0, \qquad p(0, \mathbf{x}) = \mathbf{B}_0 \cdot \nabla \times \mathbf{J}(\mathbf{x}), \qquad p_t(0, \mathbf{x}) = 0,$$

with $p(t, \mathbf{z})$ measured at all $\mathbf{z} \in \Sigma$ (a measurement surface).

One solves the inverse source problem, gets $\mathbf{B}_0 \cdot \nabla \times \mathbf{J}(\mathbf{x})$.

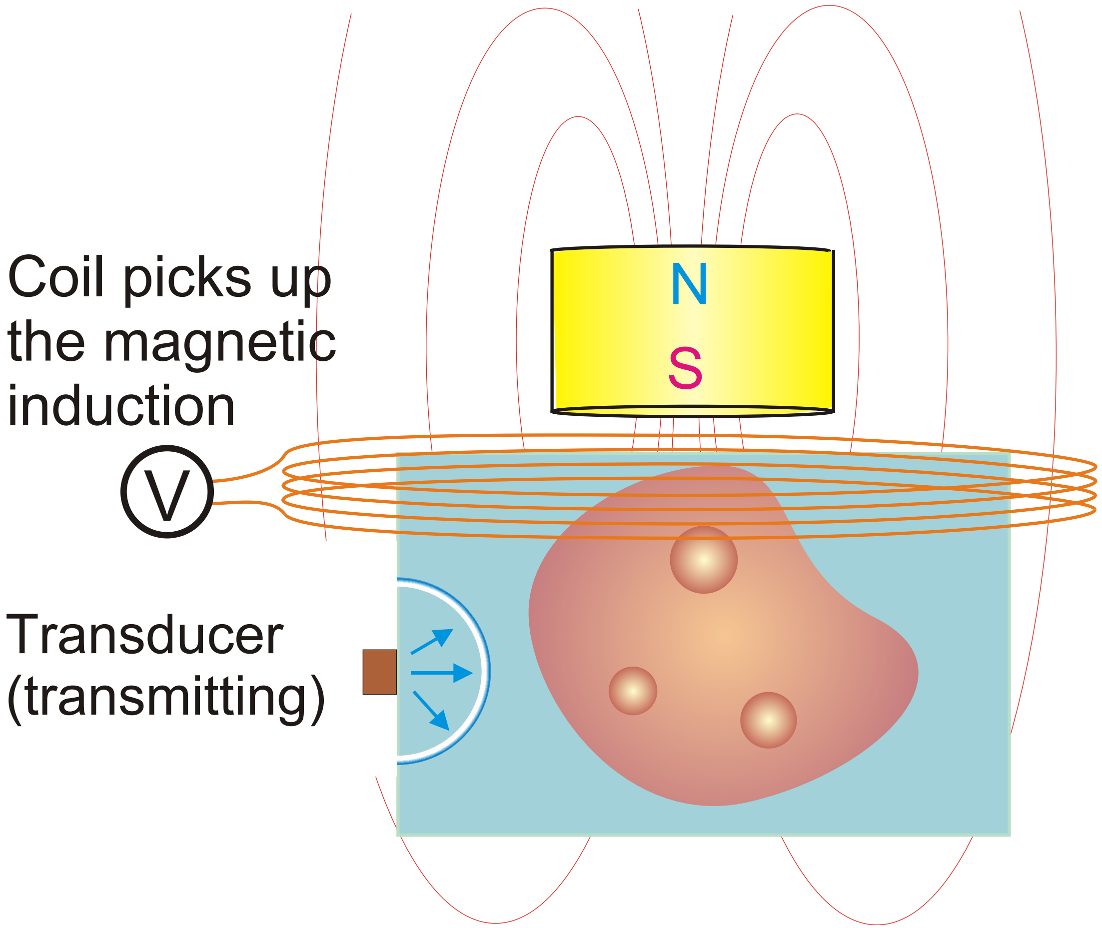
However, the eddy current $\mathbf{J}(\mathbf{x})$ is mostly circular.
It is difficult to generate several sufficiently different currents $\mathbf{J}(\mathbf{x})$.

The current $\mathbf{J}(\mathbf{x})$ must be strong = a safety issue!

Advantages: no need for the electrodes.
Could work if the surface of the object is non-conductive...
Quite popular among the experimentalists.

MAET-MI (Magnetoacoustic tomography with coil pickup)

MAET-MI is like MAT-MI in reverse:
the transducer is transmitting ultrasound, and the coil picks up small magnetic fields resulting from the Lorentz and Ohmic currents...



More on MAET-MI

Due to the reciprocity, the mathematics of MAET-MI is identical to that of MAT-MI.

One solves the inverse source problem, gets $\mathbf{B}_0 \cdot \nabla \times \mathbf{J}(\mathbf{x})$, where $\mathbf{J}(\mathbf{x})$ is now the virtual current, that would flow if the object were subjected to an inductive pulse with a certain strength...

Popular among engineers who only do modeling but not real experiments

In reality, the inductive signal is very weak. There is only one (?) succesful experimental work...

From currents to conductivity

In all magnetic modalities, one or several currents (or their components) are reconstructed/measured on the first (acoustic or MRI) step.

In MREIT and CDII one obtains $\vec{e}_3 \cdot \mathbf{B}_1$ where $\frac{1}{\mu_0}\nabla \times \mathbf{B}_1(t,\mathbf{x}) = \mathbf{J}(t,\mathbf{x})$.
With additional measurements can obtain $\mathbf{B}_1(t,\mathbf{x})$ then $\mathbf{J}(t,\mathbf{x})$.
With more measurements one can get several currents $\mathbf{J}^{(k)}(t,\mathbf{x})$.

In magnetoacoustic modalities one obtains $\mathbf{B}_0 \cdot \nabla \times \mathbf{J}(t,\mathbf{x})$. With additional measurements one can get $\nabla \times \mathbf{J}(t,\mathbf{x})$ then $\mathbf{J}(t,\mathbf{x})$.
More measurements $\to$ several currents $\mathbf{J}^{(k)}(t,\mathbf{x})$. This is cheap in MAET.

Let us consider a situation when one or several currents $\mathbf{J}^{(k)}(t,\mathbf{x})$ are known. We want to find the conductivity.

ODE for conductivity

Assume $\mathbf{J}$ is known, and so is curl $\mathbf{C} \equiv \nabla \times \mathbf{J}$.

In modalities with electrodes (MAET, MAT-CI, MREIT, CDII)

$$\mathbf{J} = \sigma \nabla u, \quad \text{and} \quad \mathbf{C} = \nabla \times \mathbf{J} = \nabla \sigma \times \nabla u = \nabla \ln \sigma \times \mathbf{J}$$

For simplicity, in 2D, let $\mathbf{C} = (0, 0, C_3)$, $\qquad \mathbf{J} = (J_1(x_1, x_2), J_2(x_1, x_2), 0)$.
Then we have a transport equation for $\ln \sigma$:

$$J_2 \frac{\partial}{\partial x_1} \ln \sigma - J_1 \frac{\partial}{\partial x_2} \ln \sigma = C_3.$$

The ODE $\frac{d}{ds}\mathbf{x}(s) = \begin{pmatrix} J_2 \\ -J_1 \end{pmatrix} (\mathbf{x}(s))$ yields lines of equal potential $\mathbf{x} = \mathbf{x}(s)$.

Conductivity $\sigma(\mathbf{x}(s_0))$ is known at a boundary point $\mathbf{x}(s_0)$. Solve the ODE

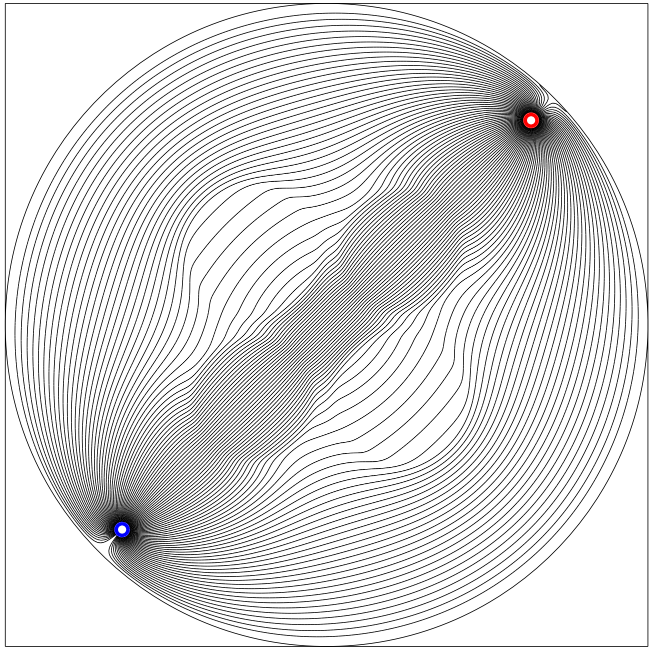
$$\frac{d \ln(\sigma(\mathbf{x}(s))}{ds} = C_3(\mathbf{x}(s)).$$

and find $\ln(\sigma(\mathbf{x}(s))$ along the characteristics $\mathbf{x} = \mathbf{x}(s)$.

MAET or MREIT or MAT-CI, 2D, one current

Conductivity (log)

Current lines



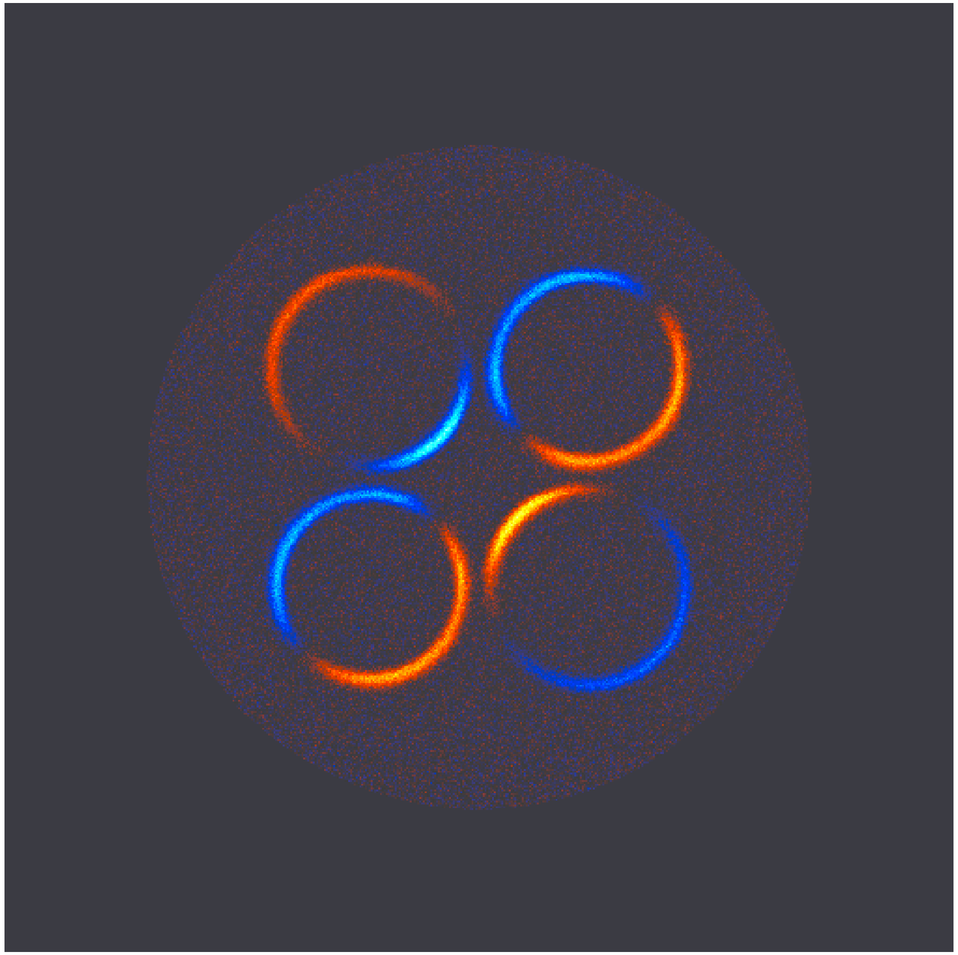
Curl

The curl is zero everywhere, except the boundaries of the inclusions!

One current, 2D simulation, adding noise

Curl

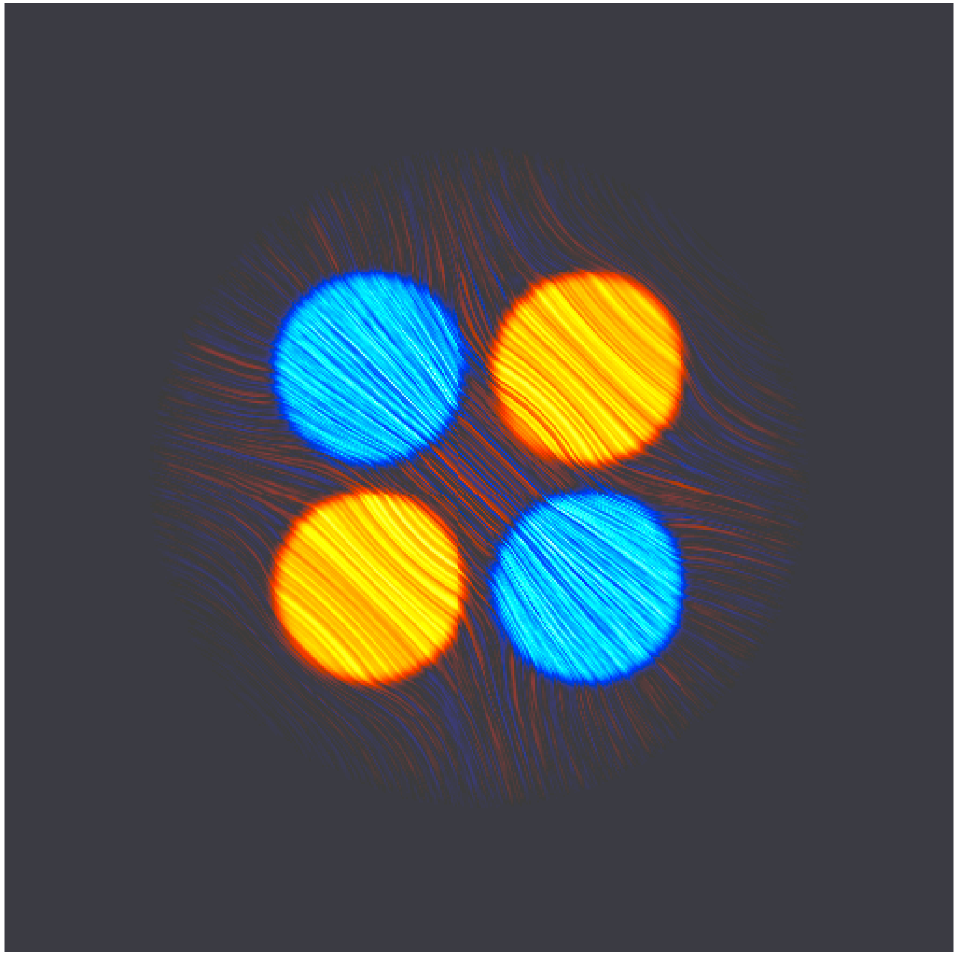
Curl + 50 % noise



Reconstruction from one current

Phantom

Reconstruction



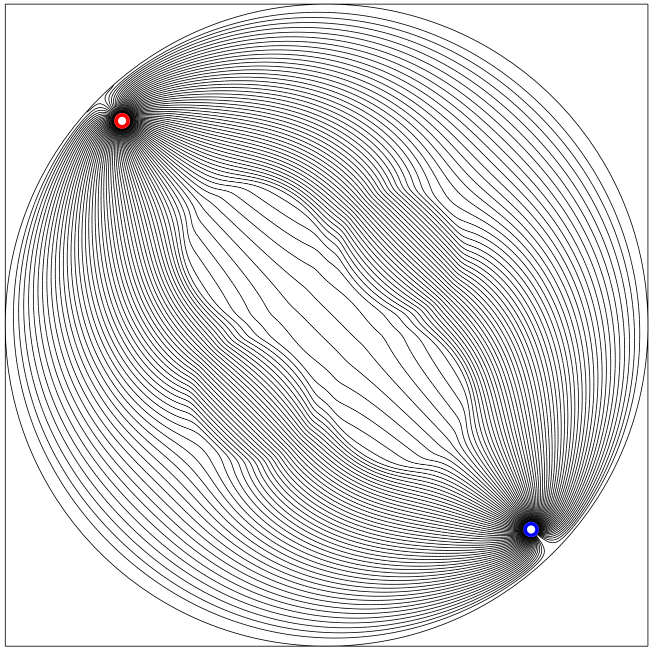
The error propagates along the characteristics = equipotential lines

The advantage of having additional currents

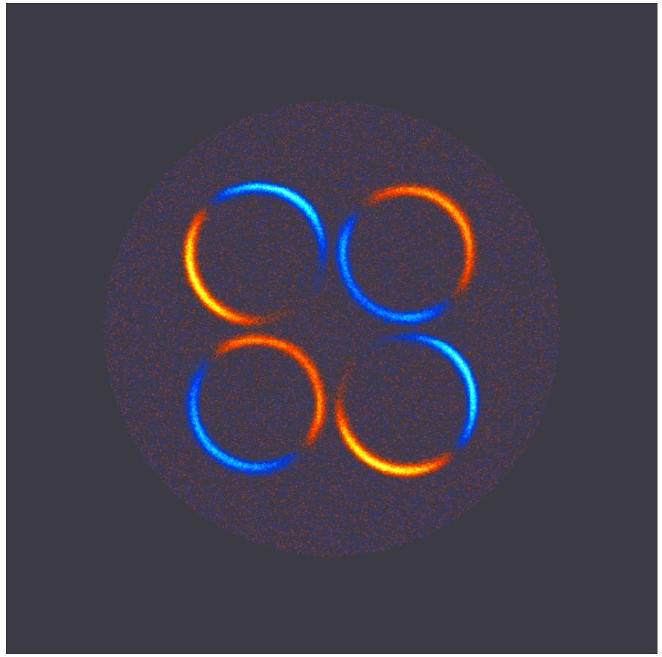
Suppose more measurements are done and a second current is known.

Conductivity (log)

Current #2 lines

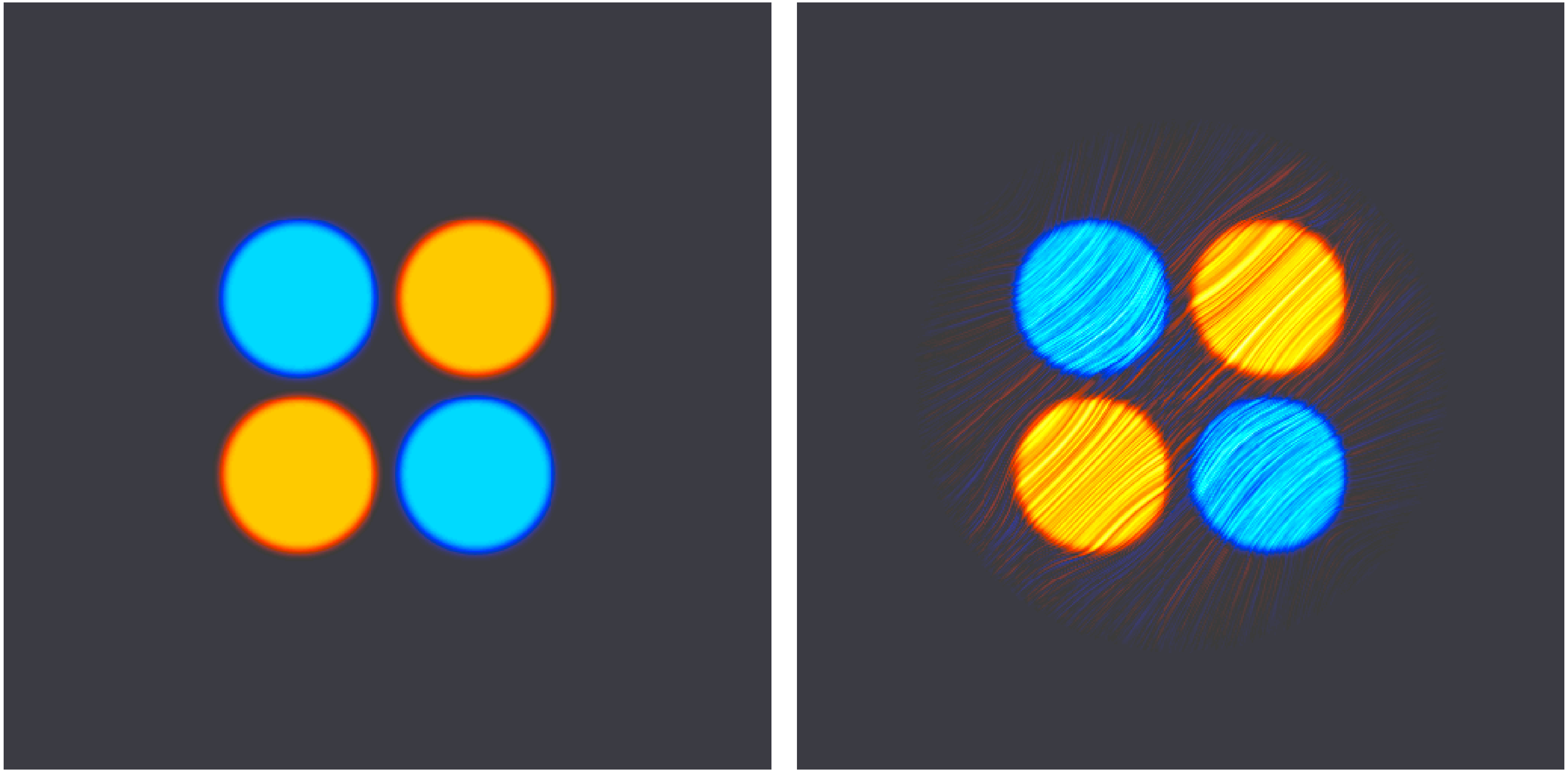


Curl #2 + 50% noise



Reconstruction from the second current

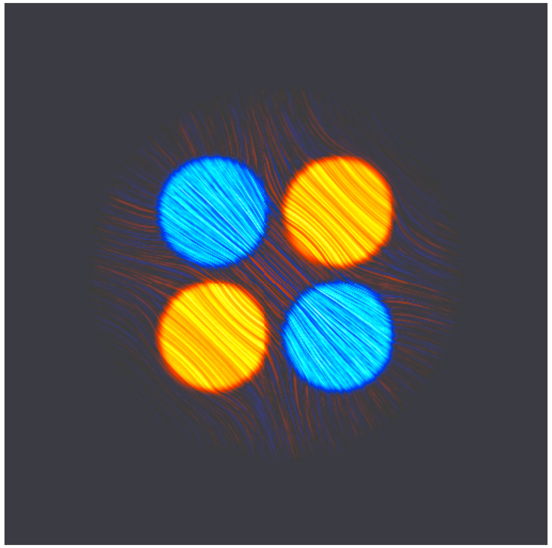
Phantom Reconstruction



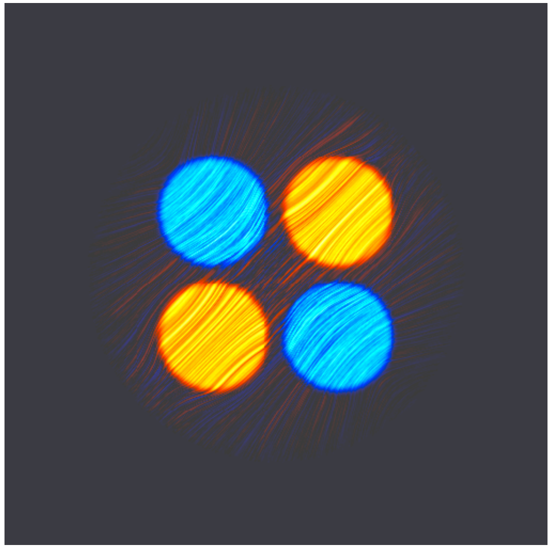
Two currents = infinitely many currents!

By linearity, any linear combination of two currents is also a current.

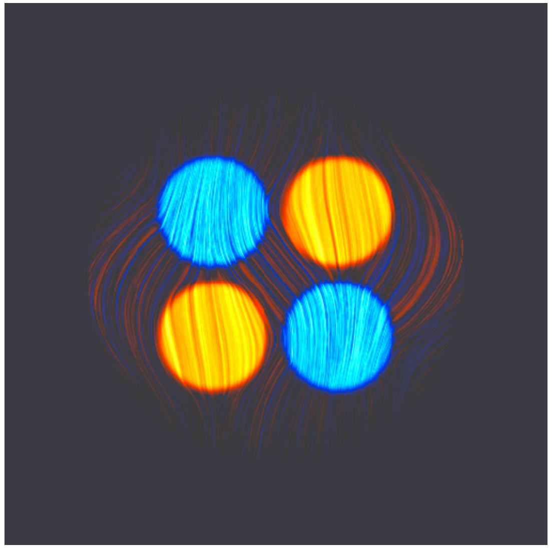
Reconstruction for current #1



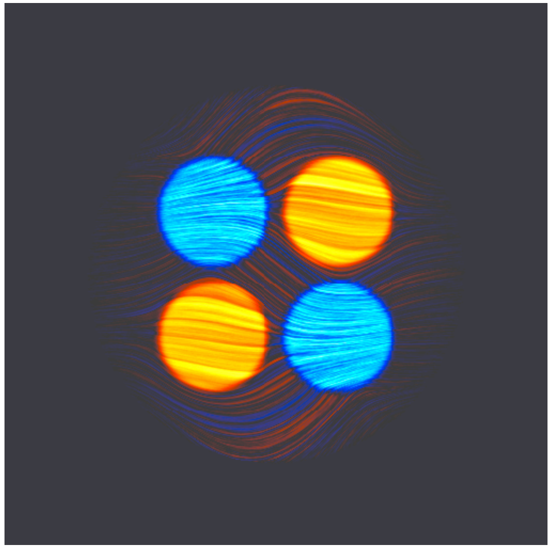
Recontruction for current #2



Reconstruction for current #1 + #2



Reconstruction for current #1 - #2

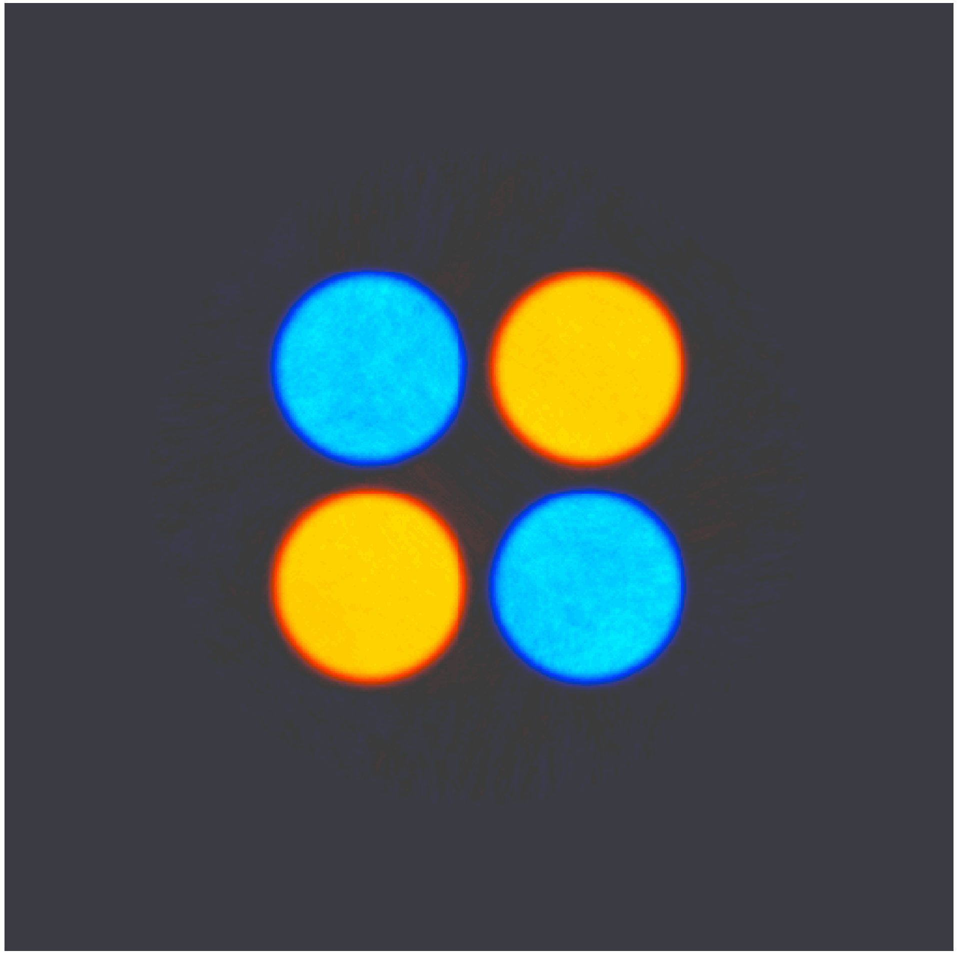


Reconstruction from two currents by linearity

128 new corrents were generated from the two measured currents,
128 reconstructions were computed. Then the average was obtained.

Phantom

Average over 128 reconstructions



Two currents as a basis

If we have two currents $\mathbf{J}^{(k)}(\mathbf{x})$ and curls $\mathbf{C}^{(k)}(\mathbf{x})$, $k = 1, 2$, then

$$\begin{cases} \nabla \ln \sigma(\mathbf{x}) \times \mathbf{J}^{(1)}(\mathbf{x}) = \mathbf{C}^{(1)}(\mathbf{x}) \\ \nabla \ln \sigma(\mathbf{x}) \times \mathbf{J}^{(2)}(\mathbf{x}) = \mathbf{C}^{(2)}(\mathbf{x}) \end{cases} .$$

At each $\mathbf{x}$, assuming the currents are not parallel, this is a system of 6 linear equations of rank at least 3.

At each $\mathbf{x}$, solve it for $\nabla \ln \sigma(\mathbf{x})$.
Compute $\mathrm{div}(\nabla \ln \sigma) = \Delta \ln \sigma$.
Solve the Poisson equation to find $\ln \sigma$.

This is much faster than averaging, and yields a similarly good result.
However, having non-parallel currents at each $\mathbf{x}$ is not guaranteed in 3D.

"A reconstruction formula and uniqueness of conductivity in MREIT using two internal current distributions"

(2004) J.-Y. Lee, Inverse Problems 20(3)

More advanced techniques

Harmonic B_Z algorithm

$\mathbf{B}_1 = (B_{1,x}, B_{1,y}, B_{1,z})(\mathbf{x})$. No object rotation. Reconstruction from $B_{1,z}$ only.

J. J. Liu *et al* "On The Convergence Of The Harmonic B_Z Algorithm In Magnetic Resonance Electrical Impedance Tomography" (2007) *SIAM J. Appl Math* **67** (5) 1259–82

J-substitution algorithm

Reconstruction from $|\mathbf{J}^1|$ and $|\mathbf{J}^2|$, $\mathbf{J}^1 \nparallel \mathbf{J}^2$

Y.J Kim *et al* "Uniqueness and convergence of conductivity image reconstruction in magnetic resonance electrical impedance tomography" (2003) *Inverse Problems* **19** 1213–25

Recovering conductivity from one current magnitude $|\mathbf{J}|$ in 2D

A. Nachman, A. Tamasan A and A. Timonov "Recovering the conductivity from a single measurement of interior data" (2007) *Inverse Problems* **25** 035014

... using $|\mathbf{J}|$ instead of $\mathbf{J}$? Why? No clear answer found ...

Example of MREIT (shamelessly borrowed)

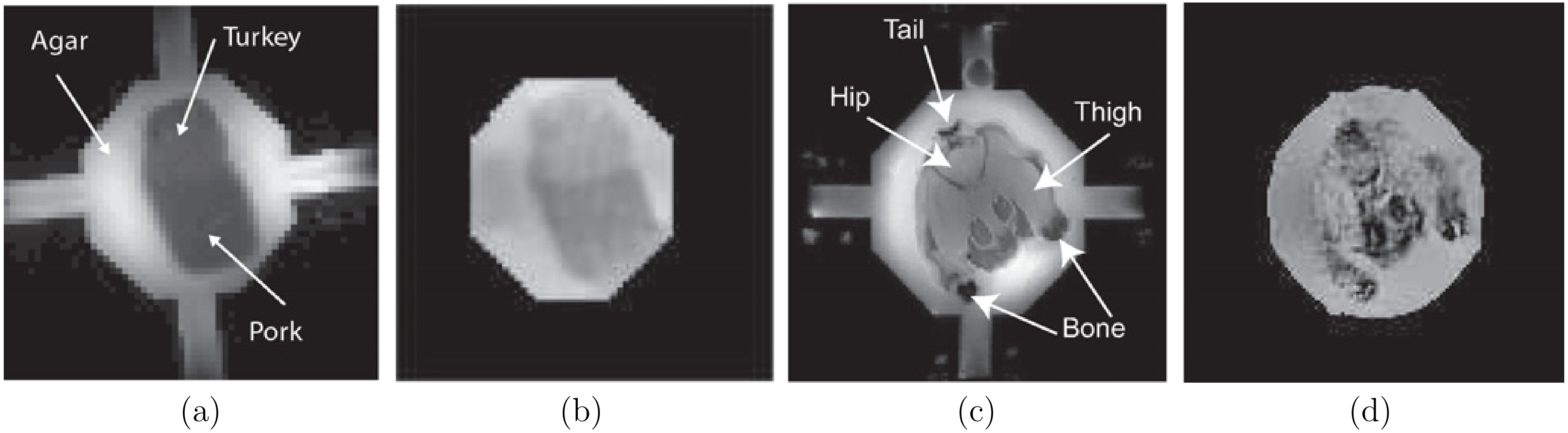


Figure 9. Biological tissue phantom imaging using an 11 T MRI scanner (Sadleir *et al* 2006). (a) MR magnitude image of a tissue phantom including chunks of turkey and pork and (b) reconstructed conductivity image of (a) using the harmonic B_z algorithm. (c) MR magnitude image of a tissue phantom including a lower part of a rat and (d) reconstructed conductivity image of (c) using the harmonic B_z algorithm.

R. Sadleir *et al* "High field MREIT: setup and tissue phantom imaging at 11T" (2006) *Physiol. Meas.* **27** S261–70

This is about as much as I presented at the conference ...