Understanding Geometry of Deep Neural Network for CT Reconstruction

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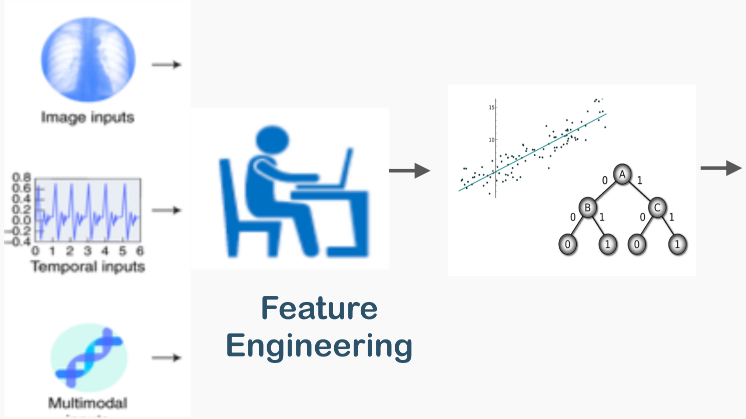
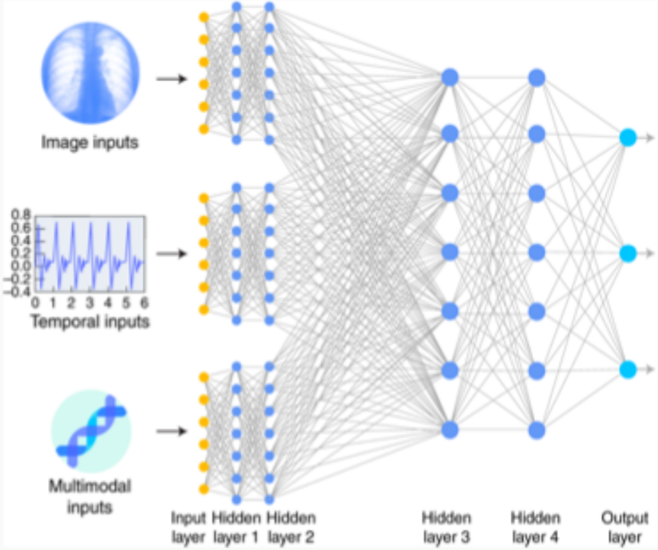
Classical Learning vs Deep Learning

Classical machine learning



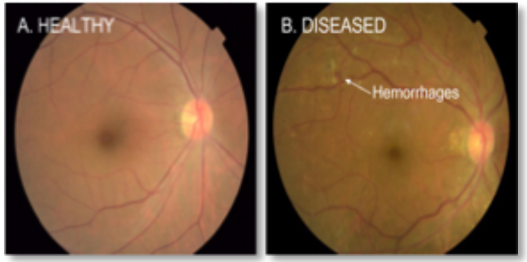
Deep learning
(no feature engineering)



Esteva et al, Nature Medicine, (2019)

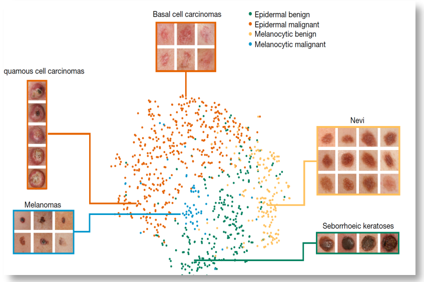
Deep Learning Era in Medical Imaging

Diabetic eye diagnosis



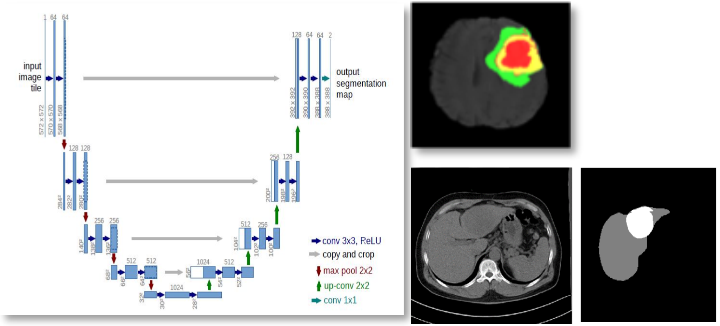
Gulshan, V. et al. JAMA (2016)

Skin Cancer diagnosis



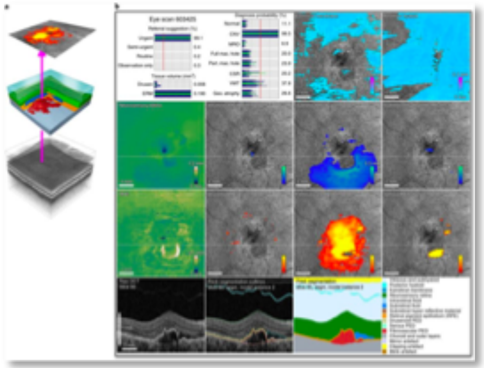
Esteva et al, Nature (2017)

Image segmentation



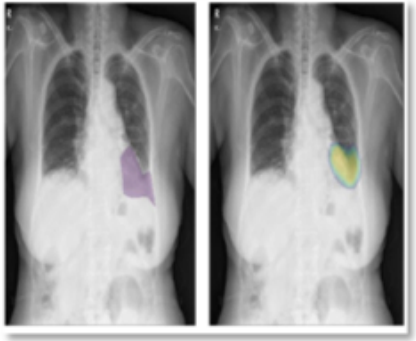
Ronneberger et al, MICCAI, 2015

OCT diagnosis



De Fauw et al, Nature Medicine (2018)

Chest X-ray



Courtesy of Kyu Hwan Jung @Vuno

Image registration

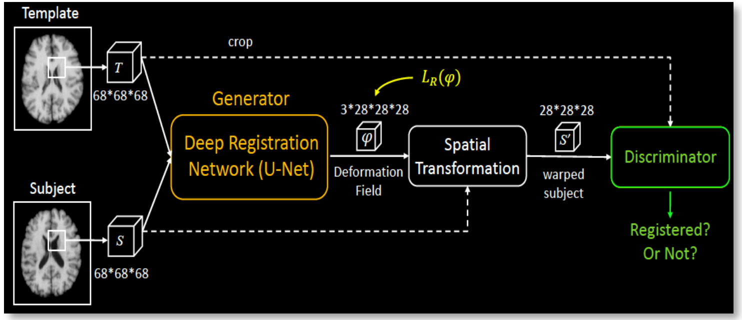
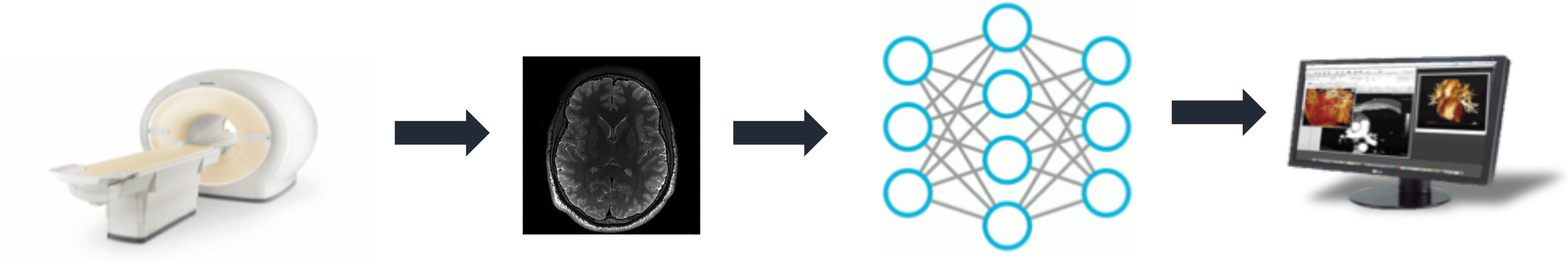


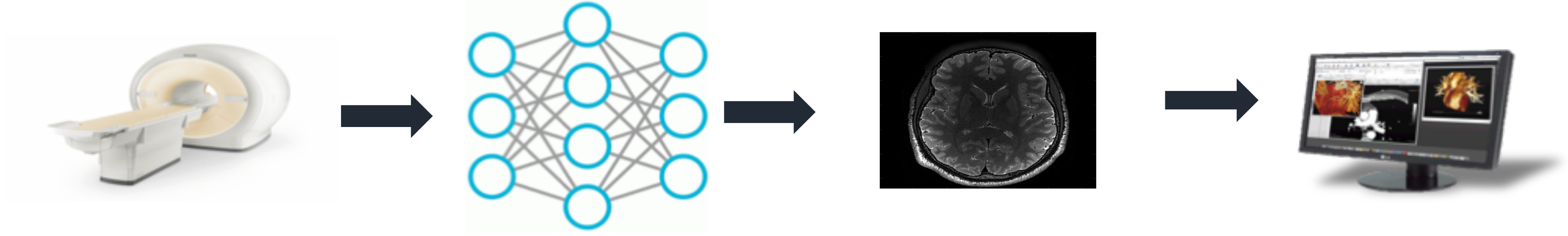
Figure courtesy of X. Cao & D. Shen

Deep Learning for Inverse Problems

Diagnosis & analysis



Focus of this talk: Reconstruction



Low Dose CT Grand Challenge

National Institute of Biomedical Imaging and Bioengineering

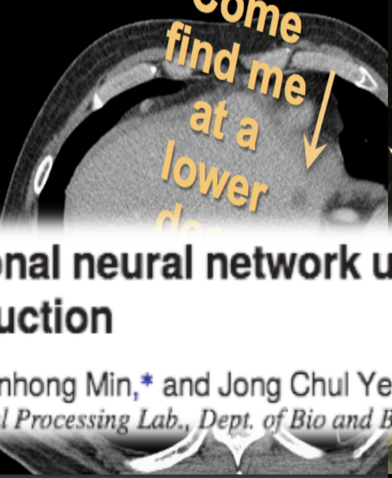
AMERICAN ASSOCIATION of PHYSICISTS IN MEDICINE

MAYO CLINIC

CT Clinical Innovation Center

- Radiologist-selected abdominal CT patient cases (10 training, 20 testing) with noise inserted to simulate lower dose acquisitions
- Projection data converted into an open format (user manual and reading tools provided)

– Apr 2016: Participants submit reconstructed images or denoised images to AAPM website



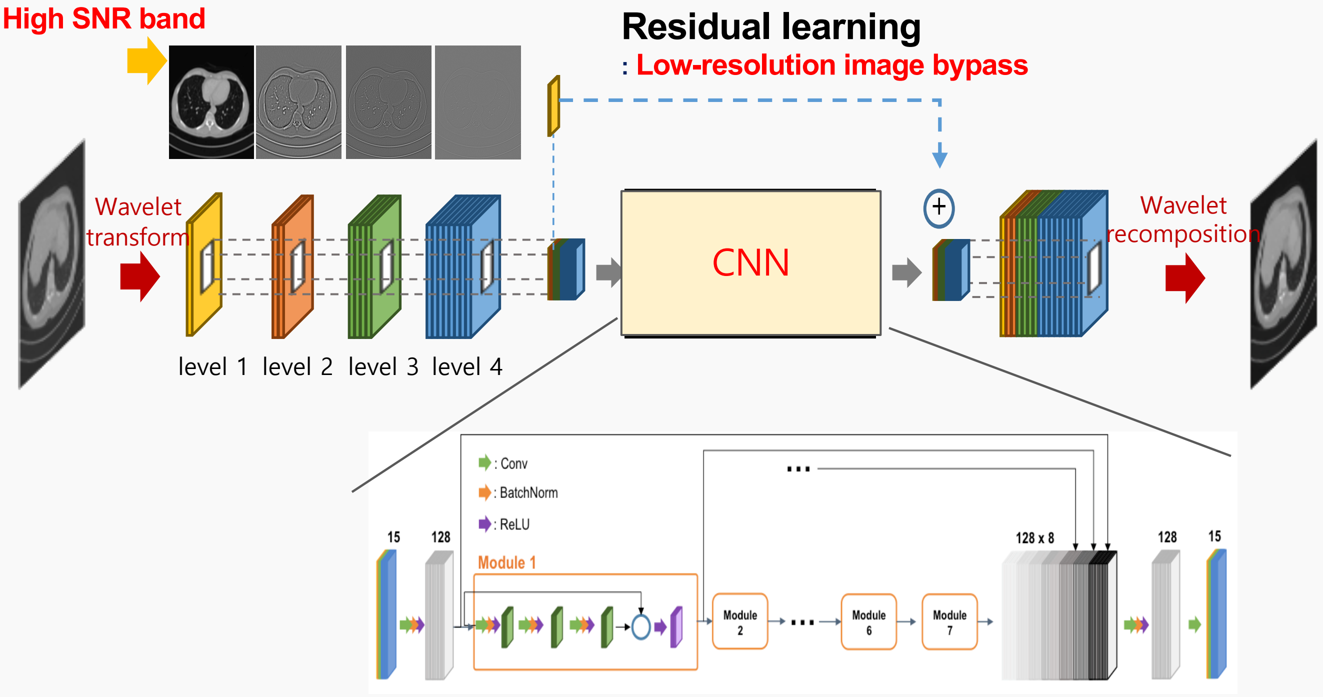
A deep convolutional neural network using directional wavelets for low-dose X-ray CT reconstruction

Med. Phys. 44 (10), October 2017

Eunhee Kang,* Junhong Min,* and Jong Chul Ye[a)]
Bio Imaging and Signal Processing Lab., Dept. of Bio and Brain Engineering, KAIST, Daejeon, Korea

www.aapm.org/GrandChallenge/LowDoseCT

AAPM-Net: 1st deep network for low-dose CT

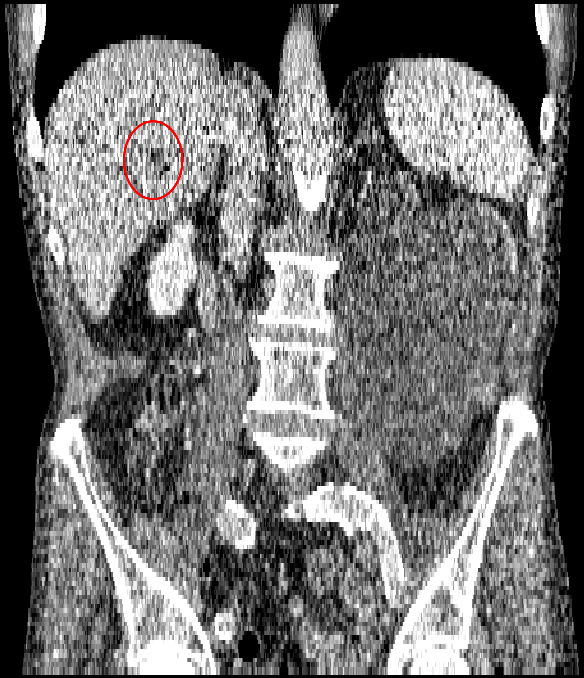


(Kang, et al, Medical Physics 44(10))

Case : L506

Full dose

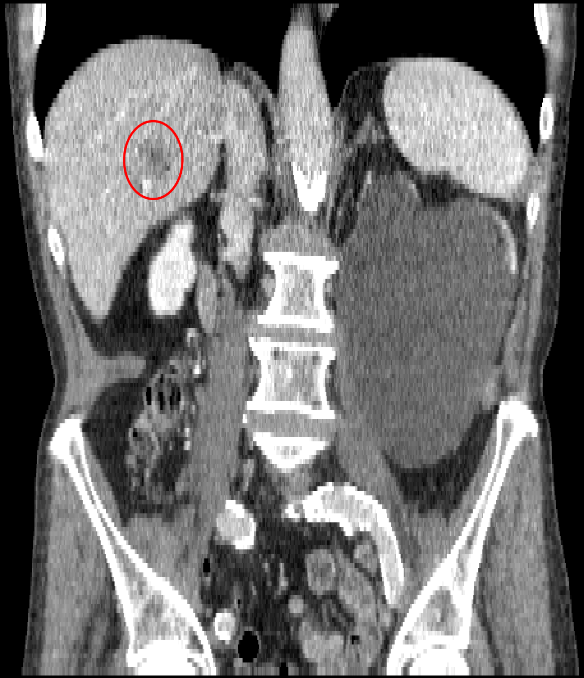
Quarter dose

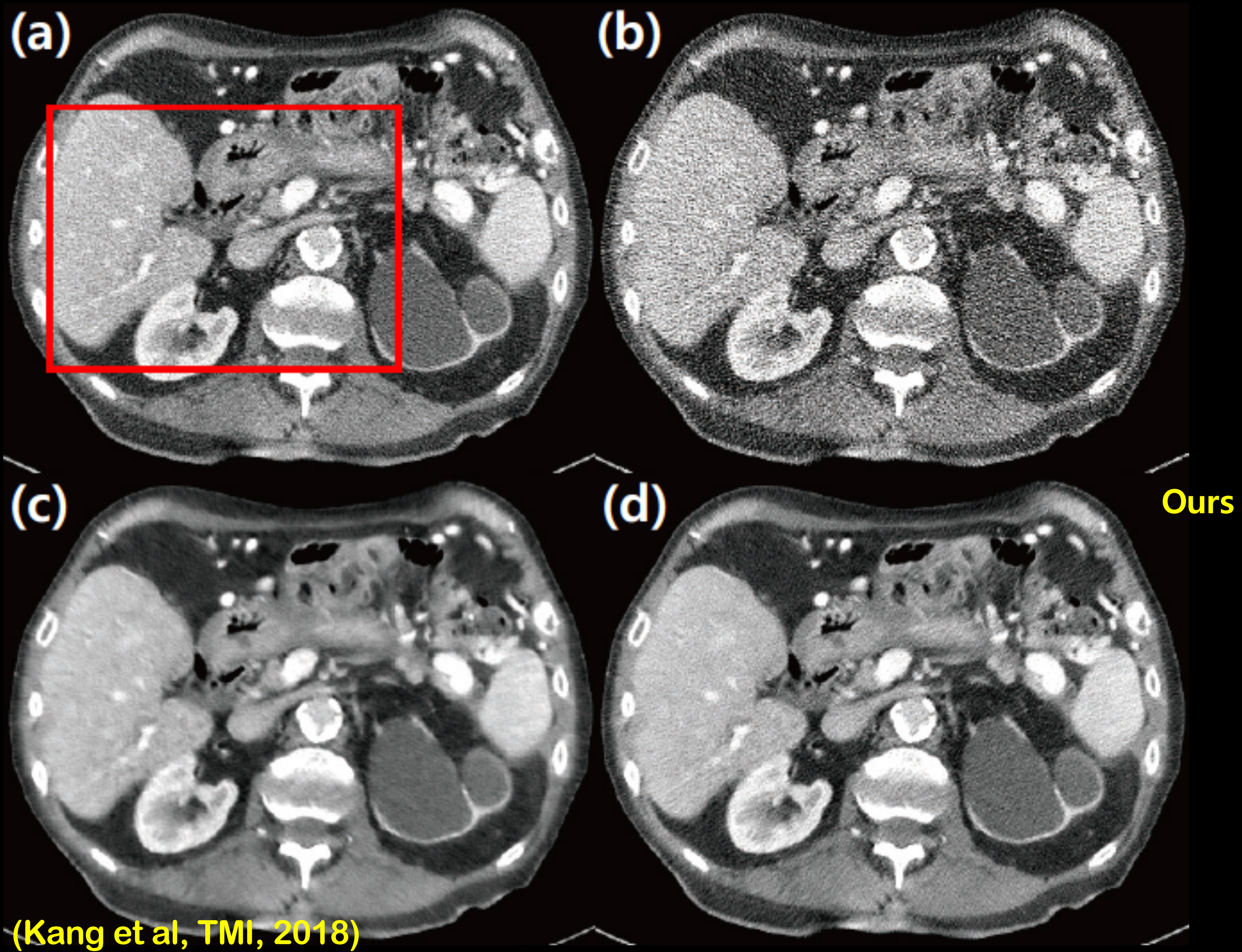


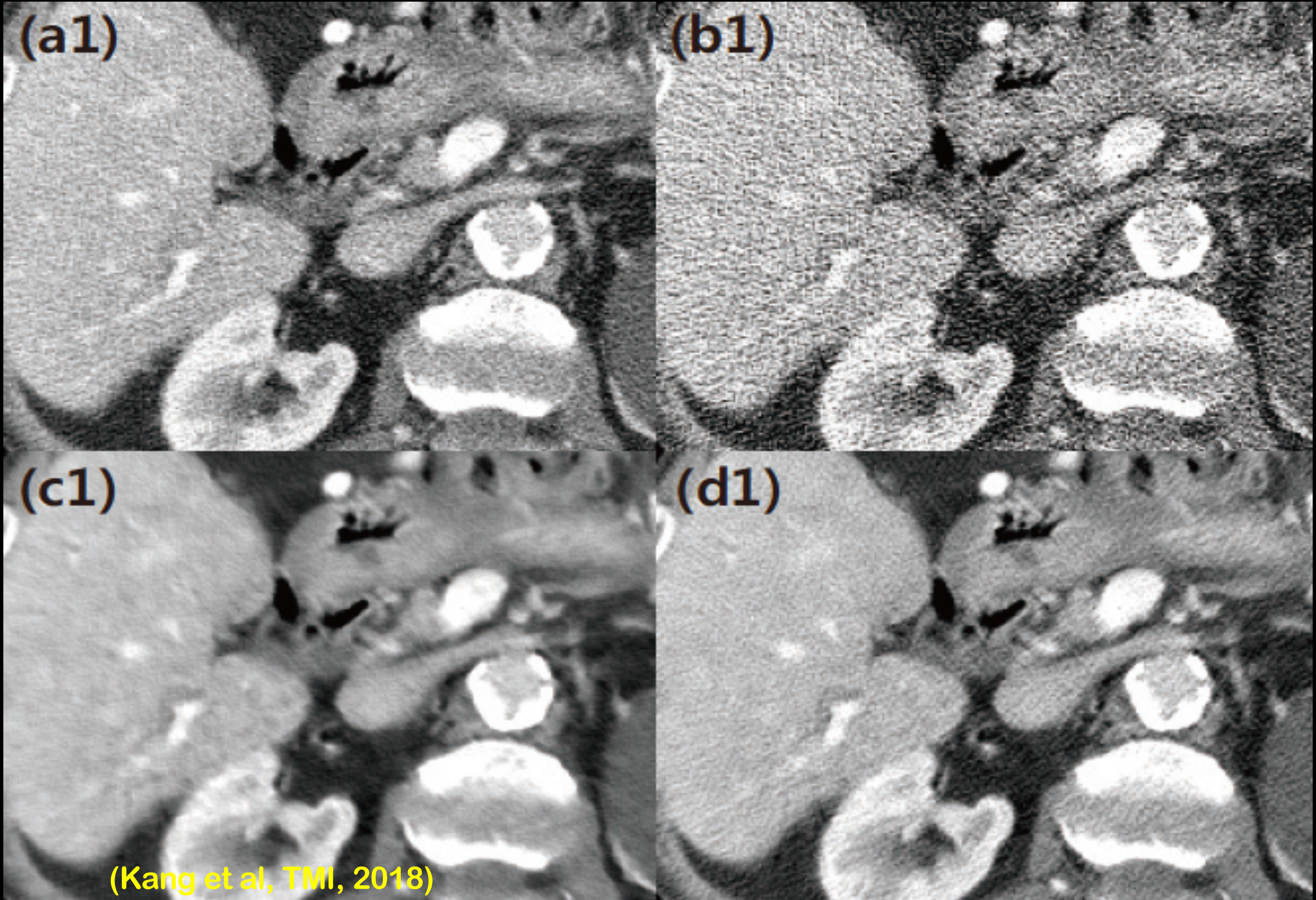
Case : L506

Full dose

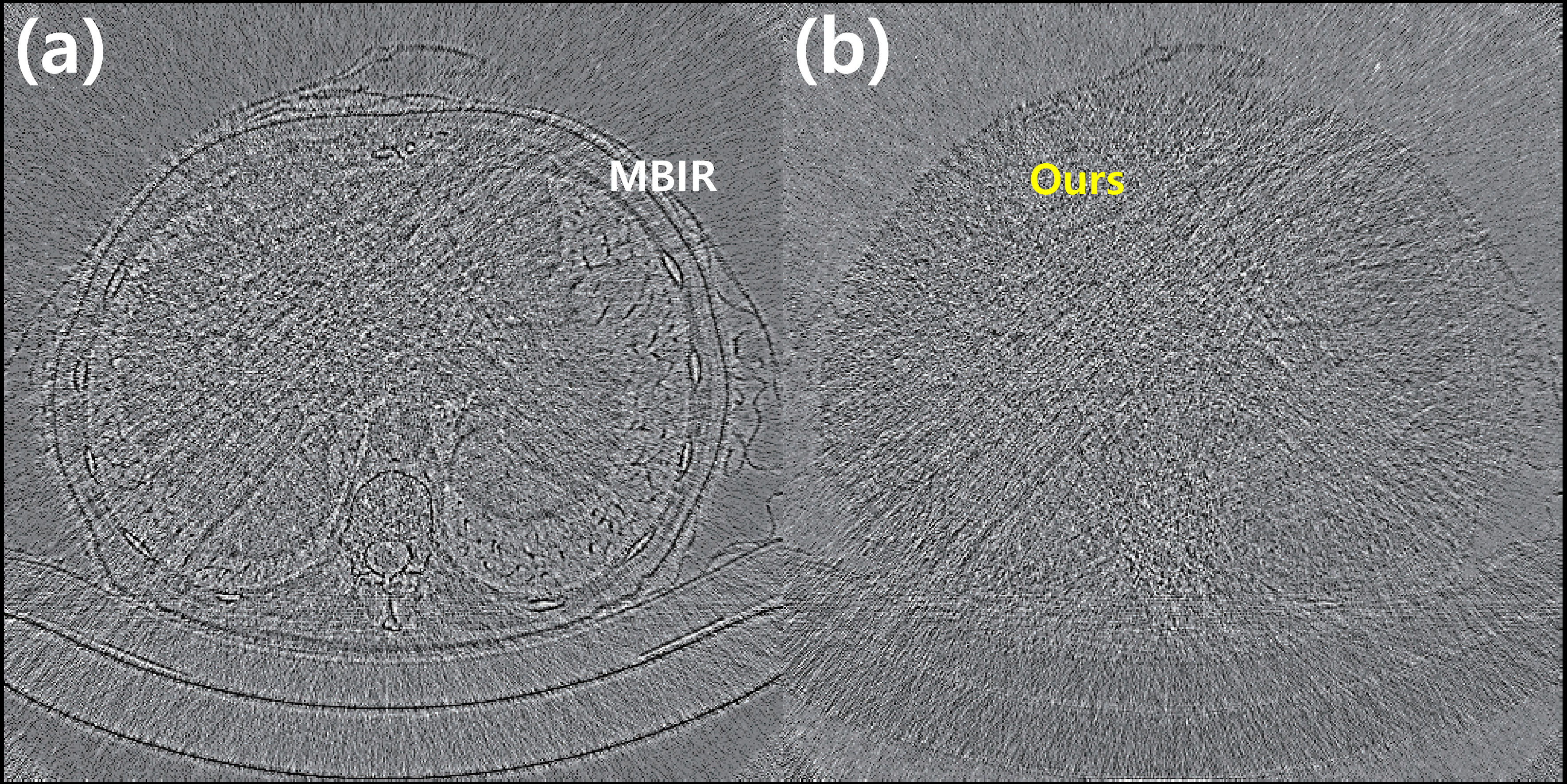
Ours from Quarter dose




(a)
(b)
(c)
(d)
Ours
(Kang et al, TMI, 2018)



Ours



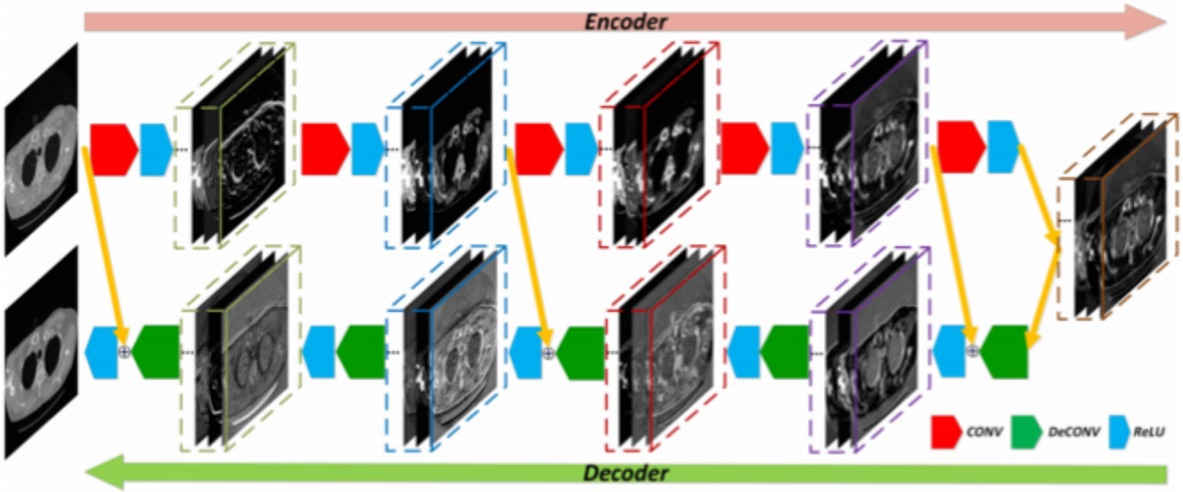
(Kang et al, TMI, 2018)

Encoder-Decoder Network for Low-Dose CT

2524 IEEE TRANSACTIONS ON MEDICAL IMAGING, VOL. 36, NO. 12, DECEMBER 2017

Low-Dose CT With a Residual Encoder-Decoder Convolutional Neural Network

Hu Chen, Yi Zhang, *Member, IEEE*, Mannudeep K. Kalra, Feng Lin, Yang Chen,
Peixi Liao, Jiliu Zhou, *Senior Member, IEEE*, and Ge Wang, *Fellow, IEEE*

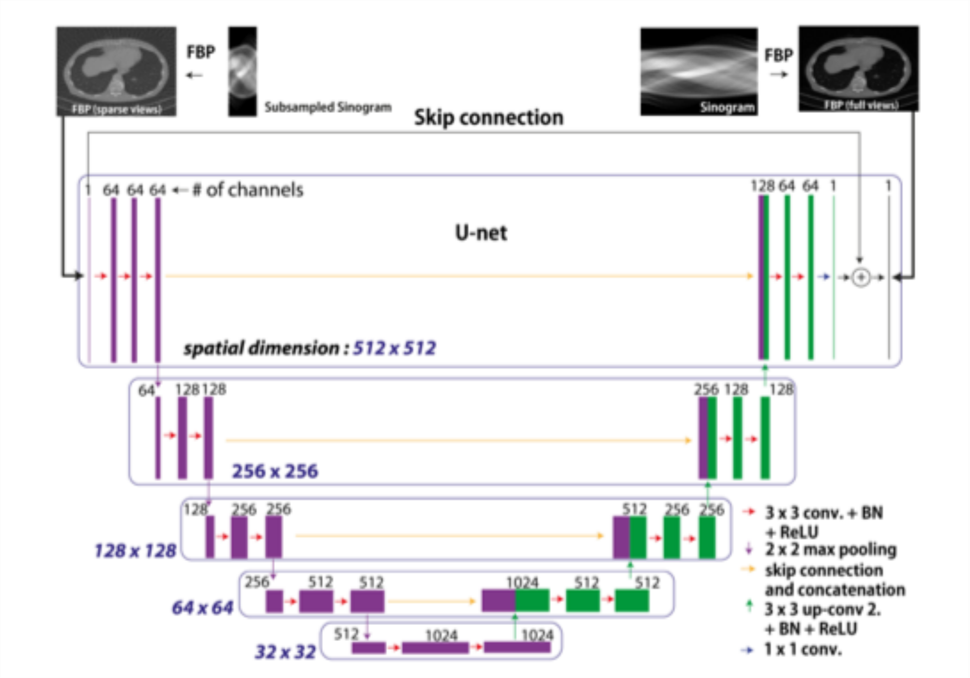


FBPConvNet

IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 26, NO. 9, SEPTEMBER 2017 4509

Deep Convolutional Neural Network for Inverse Problems in Imaging

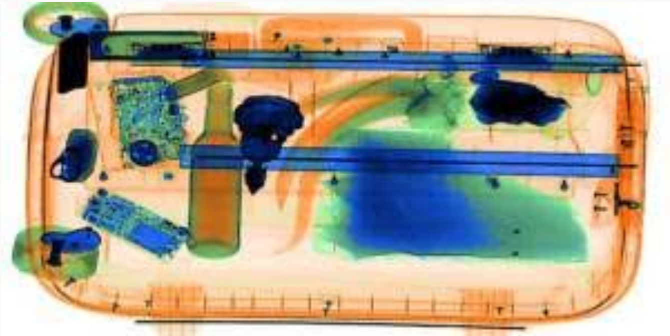
Kyong Hwan Jin, Michael T. McCann, *Member, IEEE*, Emmanuel Froustey, and Michael Unser, *Fellow, IEEE*

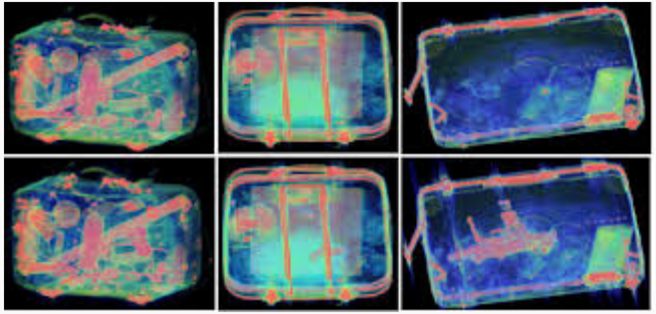


Extreme Sparse View CT

Han et al, arXiv preprint arXiv:1712.10248, (2017); CT Meeting (2017)

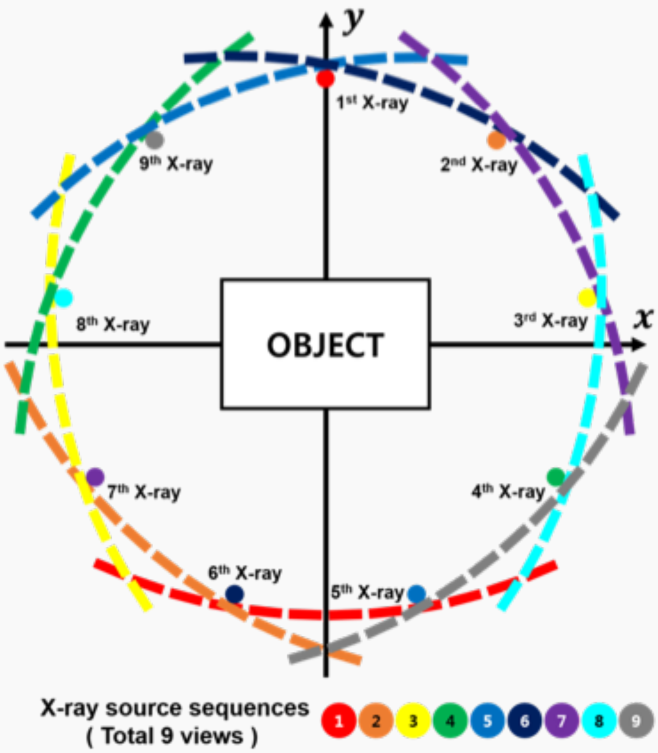
Stationary CT Carry on baggage scanner



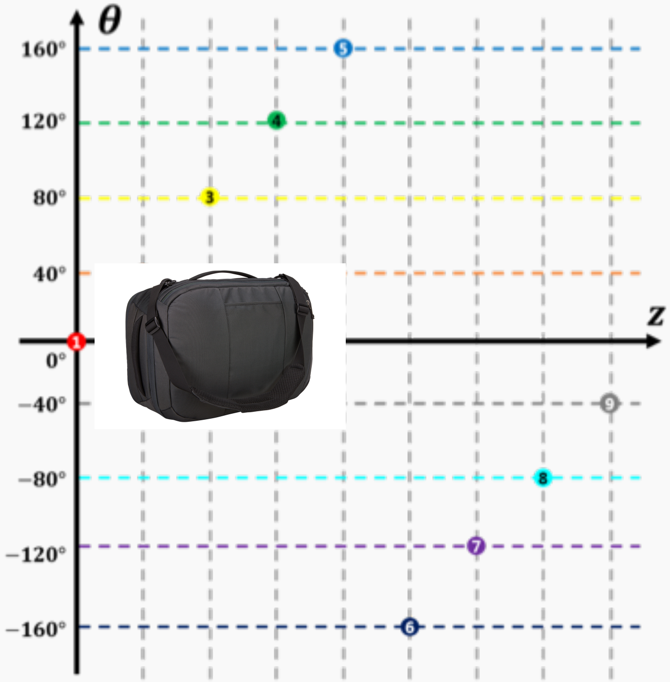


Figures from internet

Source/detector configuration

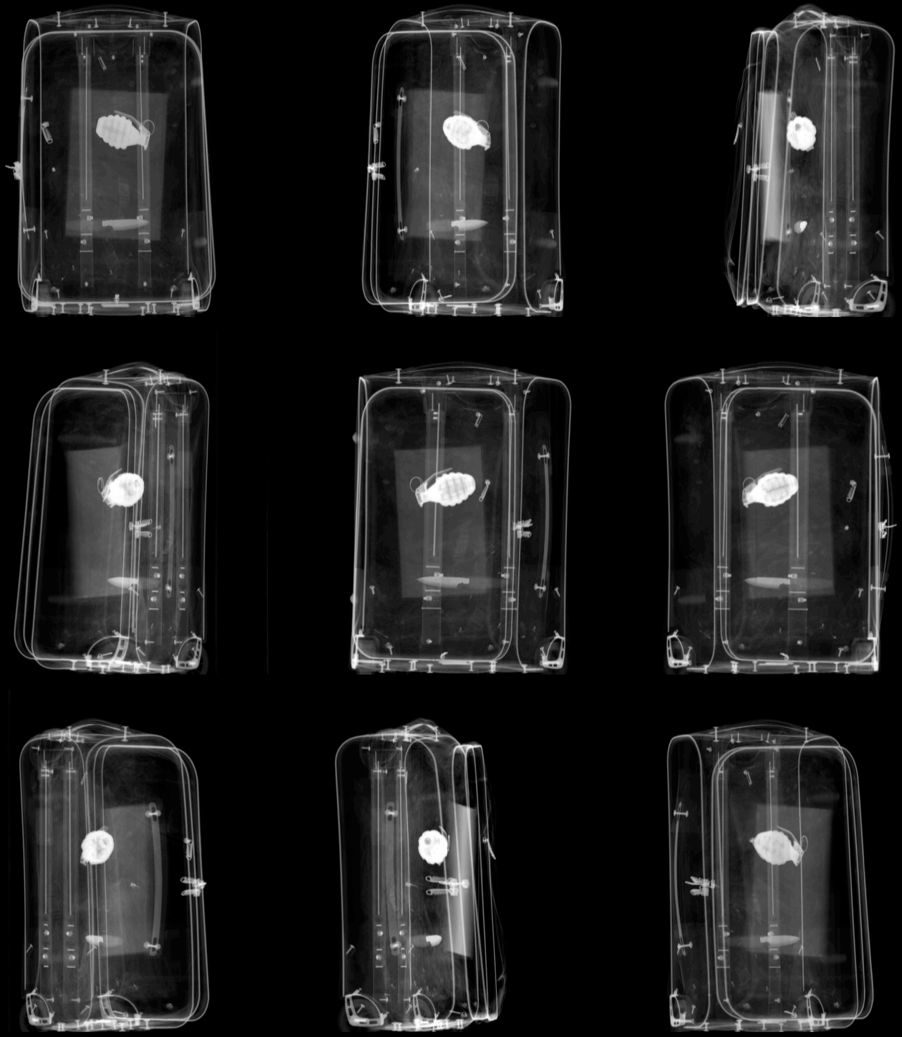


(a)



(b)

Han et al, arXiv preprint arXiv:1712.10248, (2017); CT Meeting (2017)

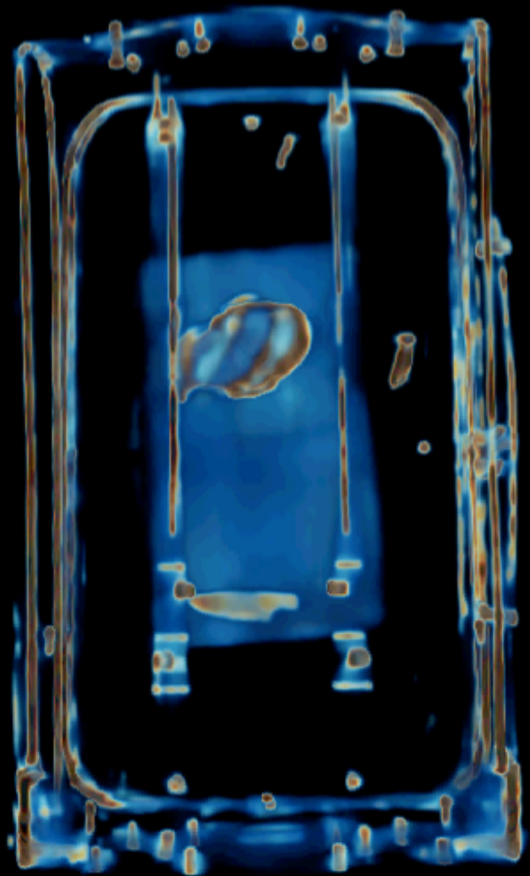


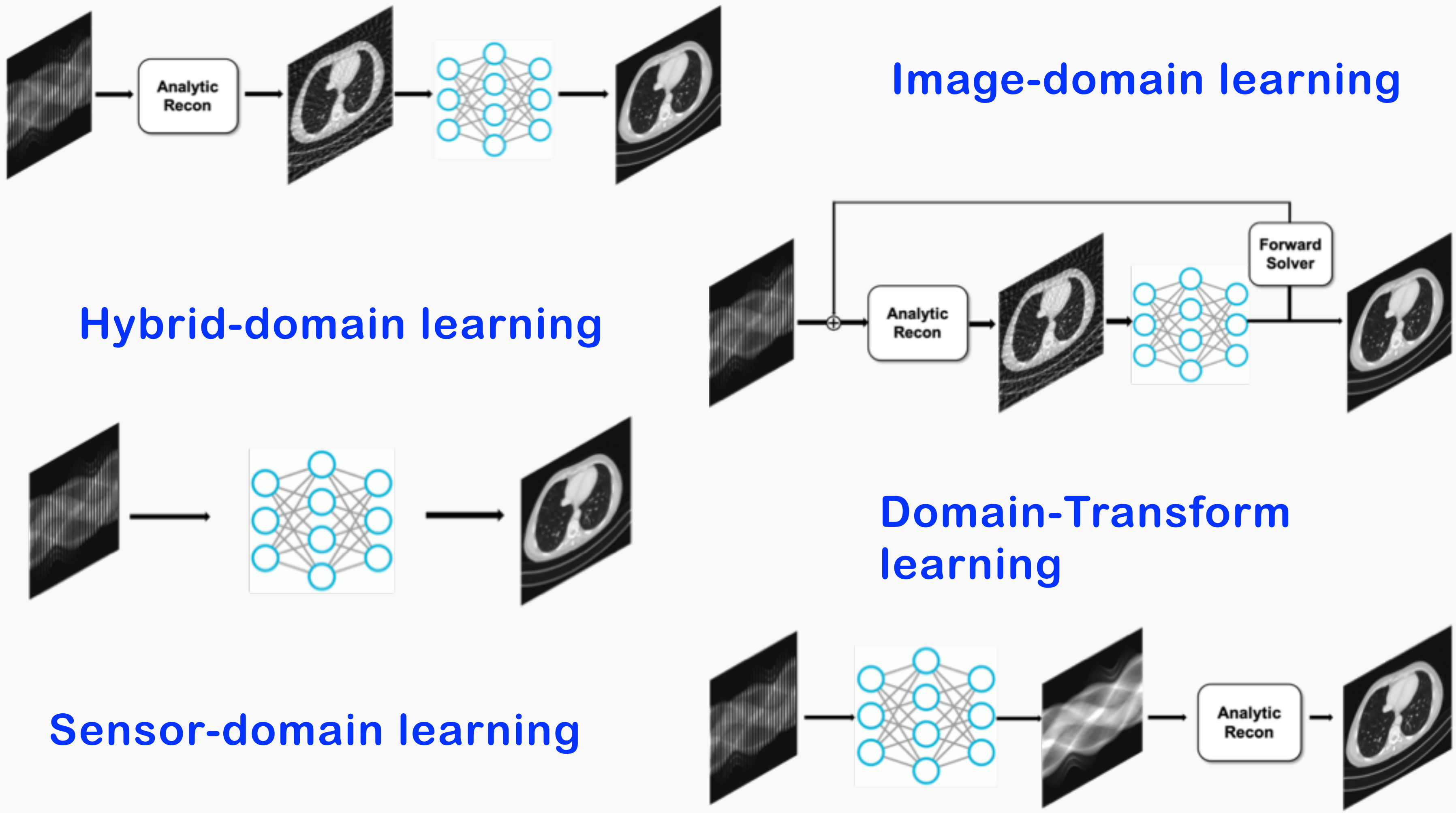
FBP

TV

Deep Learning

Han et al, CT meetings, 2018





Analytic Recon
Image-domain learning
Forward Solver
Hybrid-domain learning
Analytic Recon
Domain-Transform learning
Sensor-domain learning
Analytic Recon

Why so popular this time ?

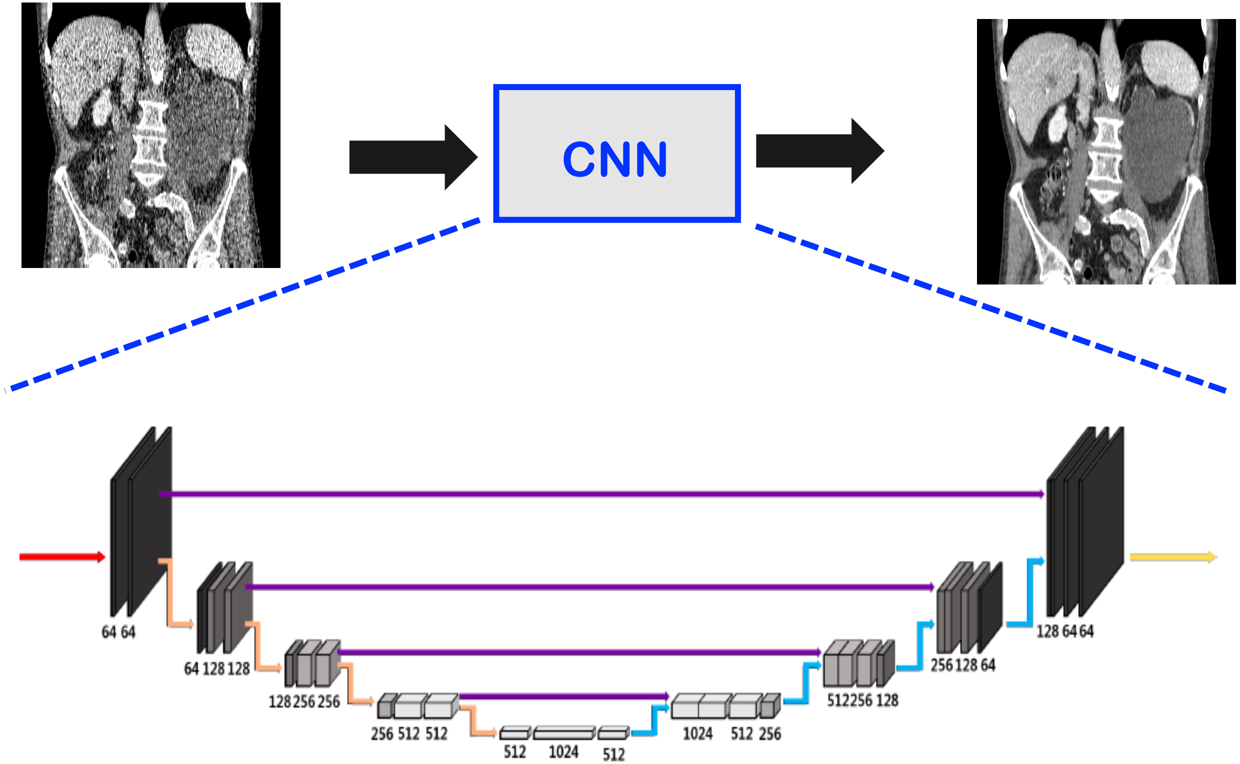
- ❑ **Accuracy**: high quality recon > CS
- ❑ **Fast** reconstruction time
- ❑ **Business model:** vendor-driven training
- ❑ **Interpretable** models
- ❑ **Flexibility:** more than recon

DOES IT CREATE ANY ARTIFICIAL FEATURES ?

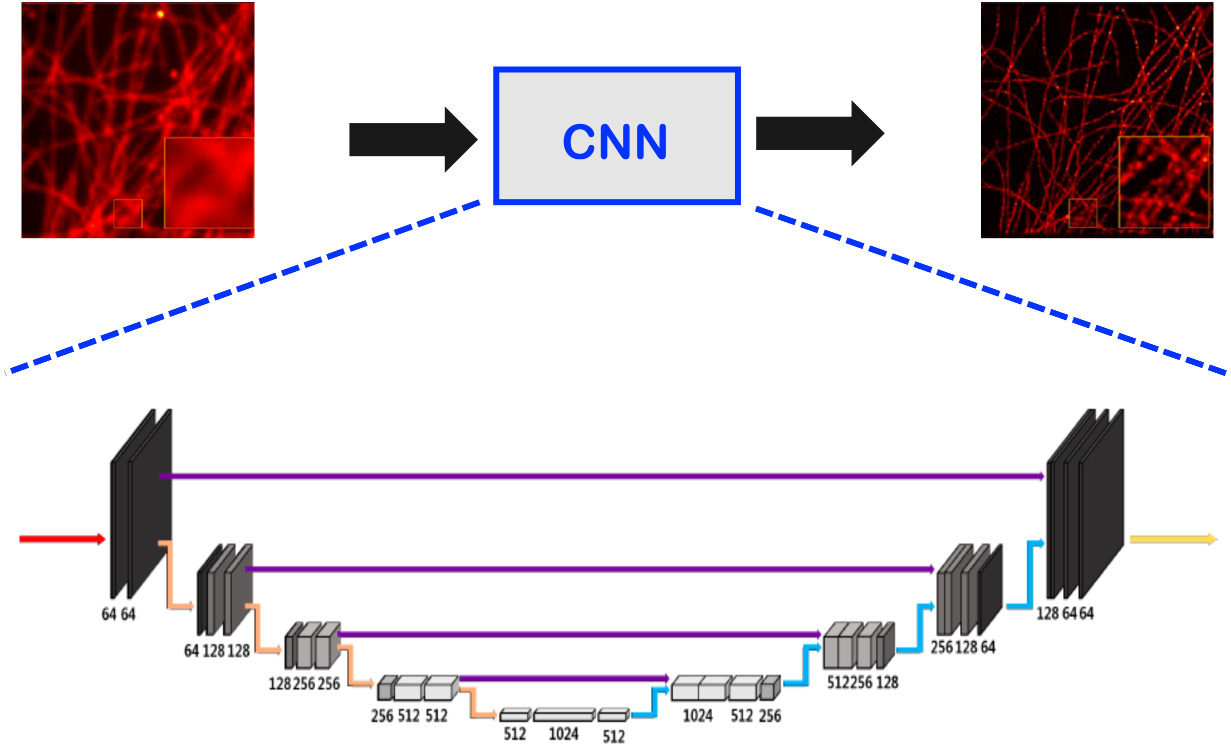
Understanding Geometry of CNN

Ye et al, SIAM J. Imaging Sciences, 2018; Ye et al, ICML, 2019

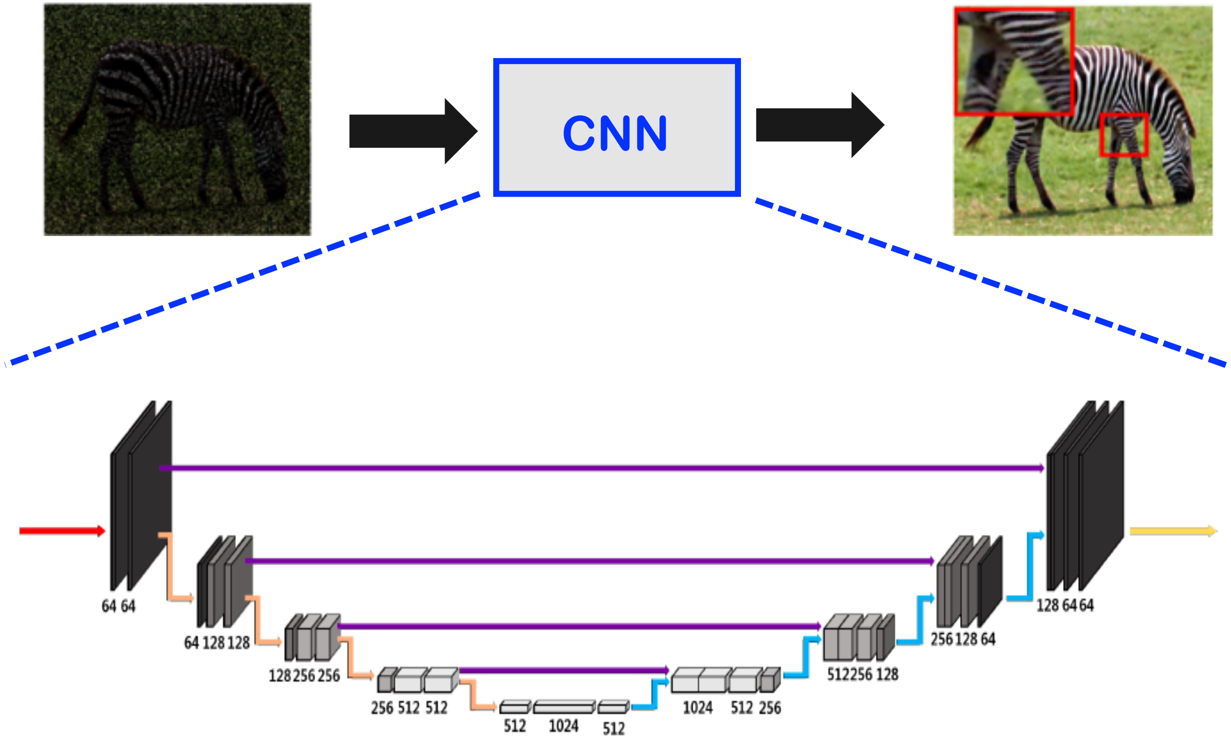
Encoder-Decoder CNN for Inverse Problems



Encoder-Decoder CNN for Inverse Problems



Encoder-Decoder CNN for Inverse Problems



Successful applications to various inverse problems

Why Same Architecture Works for Different Inverse Problems ?

Classical Methods for Inverse Problems

Step 1: Signal Representation

$$x = \sum_i \overbrace{\langle b_i , x \rangle}^{\text{coefficients}} \tilde{b}_i$$

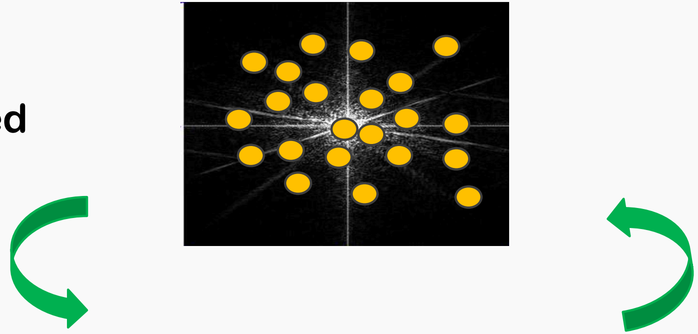
Analysis frame

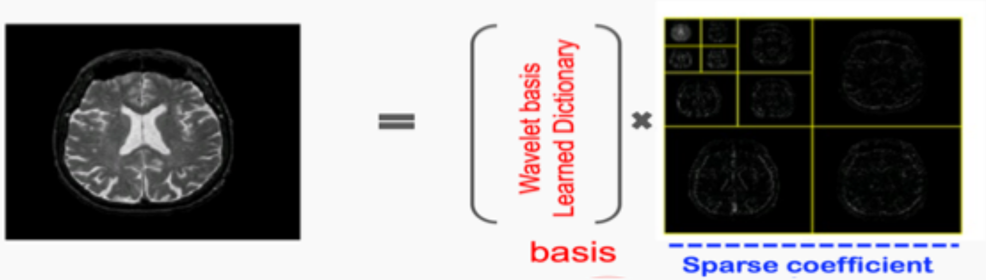
Synthesis frame

Classical Methods for Inverse Problems

Step 2: Basis Search by Optimization

Eg. Compressed Sensing



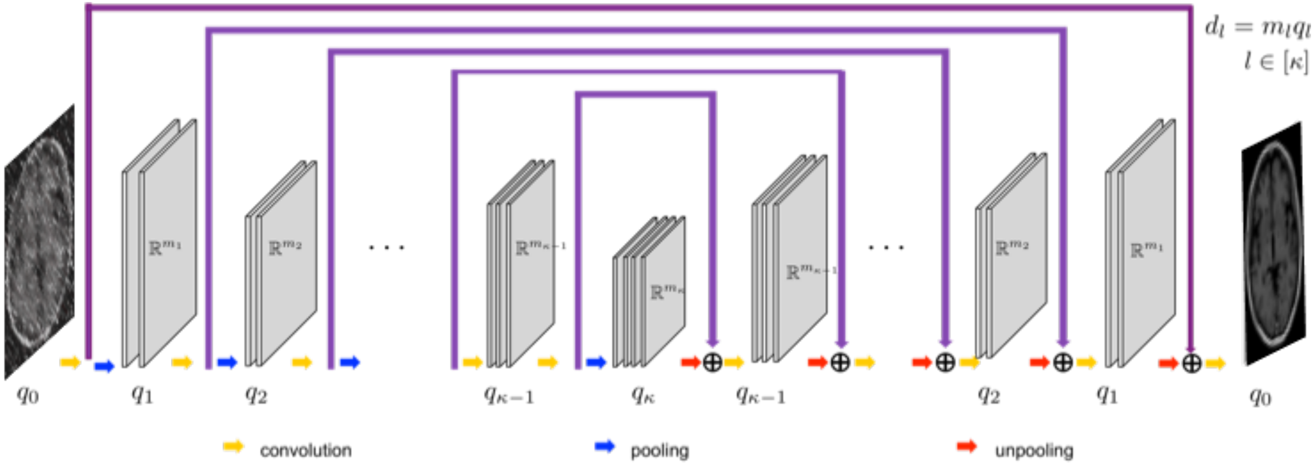


$$x = \sum_i \tilde{b}_i \langle b_i , x \rangle$$

Why do They Look so Different ?
Any Link between Them ?

Our Theoretical Findings

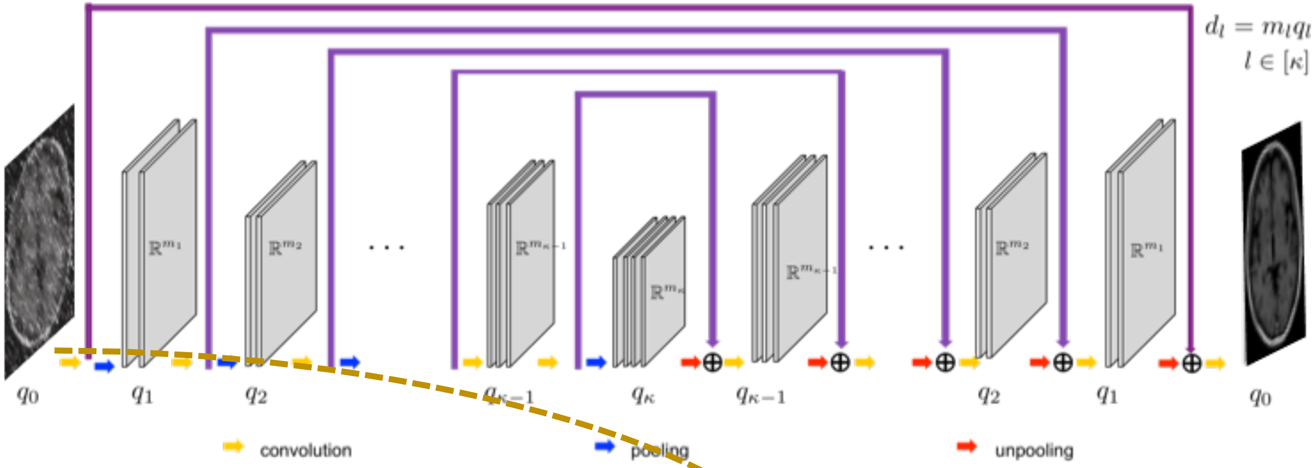
Ye et al, SIIMS, 2018; Ye et al, ICML, 2019



$$y = \sum_i \langle b_i(x), x \rangle \tilde{b}_i(x)$$

Our Theoretical Findings

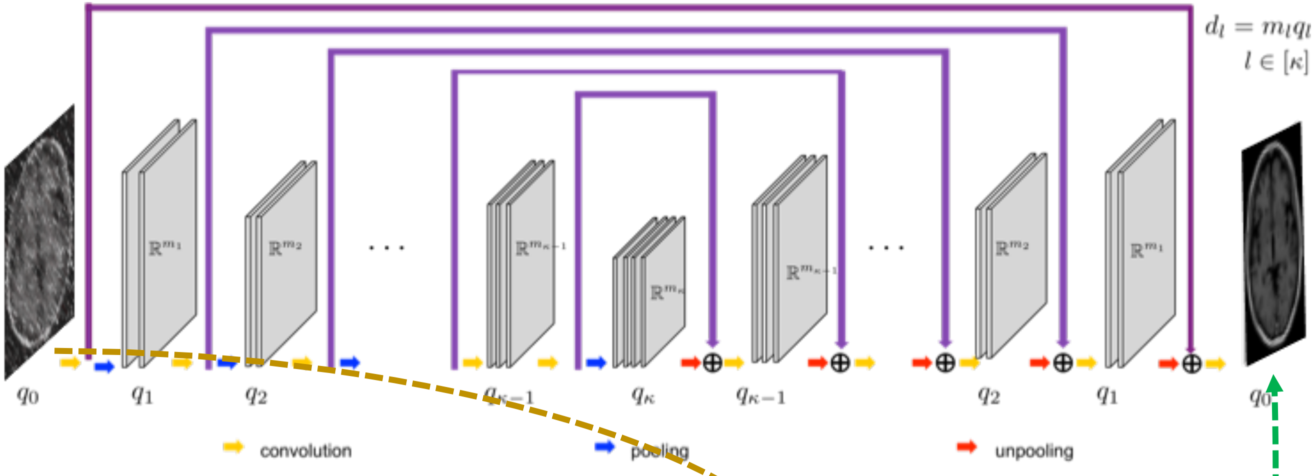
Ye et al, SIIMS, 2018; Ye et al, ICML, 2019



$$y = \sum_i \langle b_i(x), x \rangle \tilde{b}_i(x)$$

Our Theoretical Findings

Ye et al, SIIMS, 2018; Ye et al, ICML, 2019

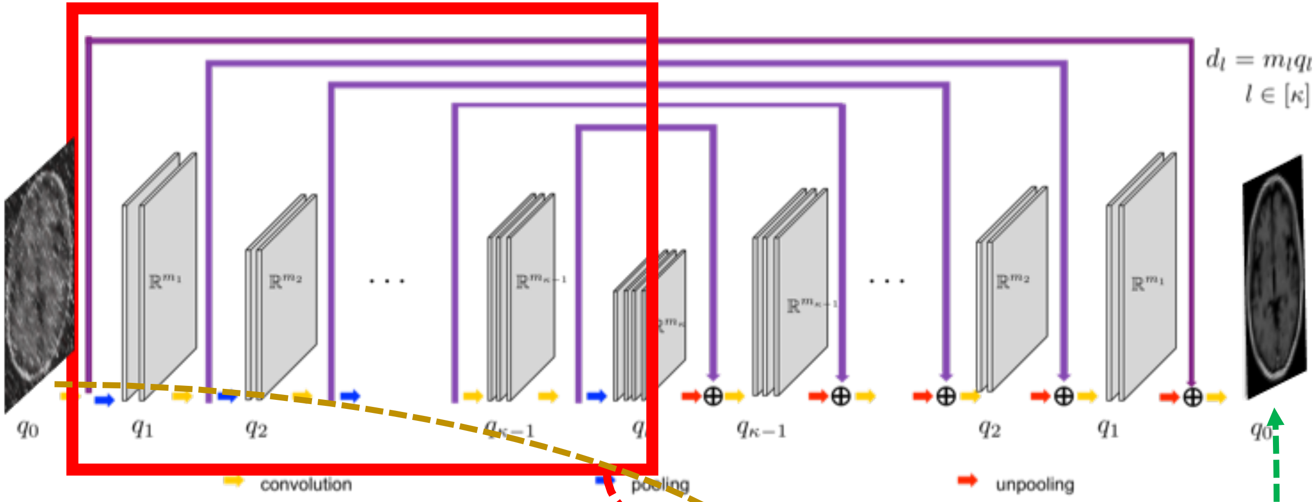


$$y = \sum_i \langle b_i(x), x \rangle \tilde{b}_i(x)$$

Our Theoretical Findings

Ye et al, SIIMS, 2018; Ye et al, ICML, 2019

Encoder

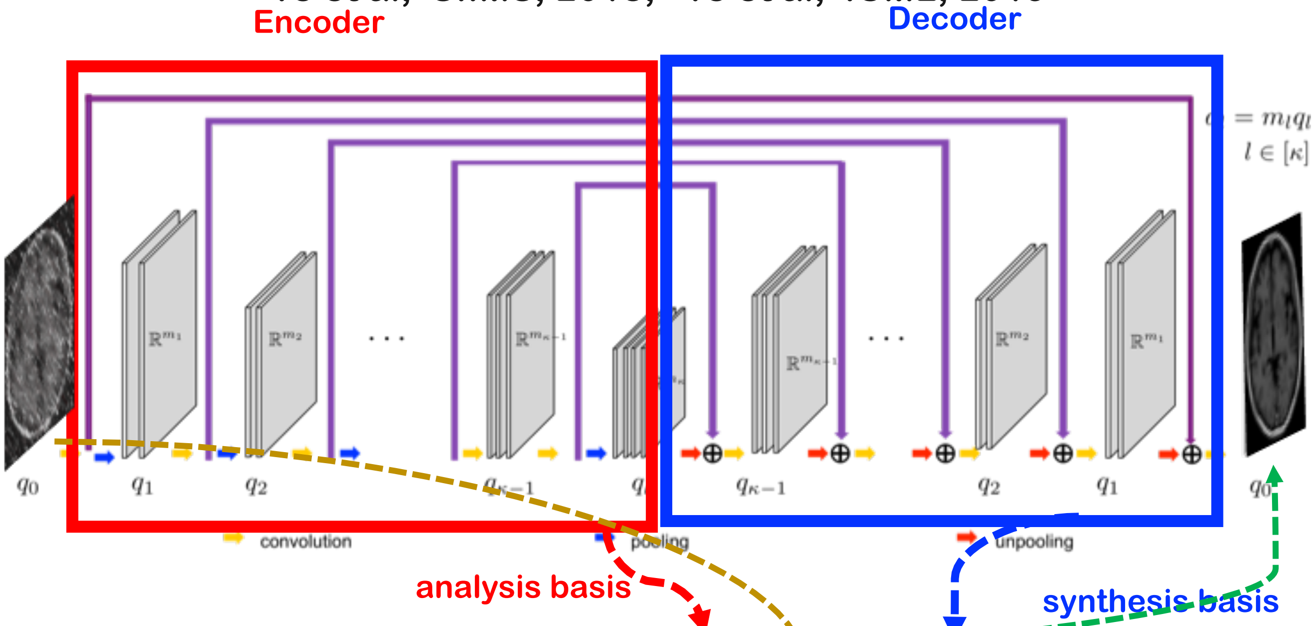


analysis basis

$$y = \sum_i \langle b_i(x), x \rangle \tilde{b}_i(x)$$

Our Theoretical Findings

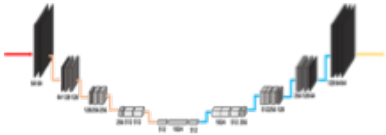
Ye et al, SIIMS, 2018; Ye et al, ICML, 2019



$$y = \sum_i \langle b_i(x), x \rangle \tilde{b}_i(x)$$

Linear Encoder-Decoder (ED) CNN

$$y = \tilde{B} B^\top x = \sum_i \langle x, b_i \rangle \tilde{b}_i$$



$$\begin{aligned} B &= E^1 E^2 \cdots E^\kappa, \\ \tilde{B} &= D^1 D^2 \cdots D^\kappa \end{aligned}$$

pooling

un-pooling

$$E^l = \begin{bmatrix} \Phi^l \circledast \psi^l_{1,1} & \cdots & \Phi^l \circledast \psi^l_{q_l,1} \\ \vdots & \ddots & \vdots \\ \Phi^l \circledast \psi^l_{1,q_{l-1}} & \cdots & \Phi^l \circledast \psi^l_{q_l,q_{l-1}} \end{bmatrix} \qquad D^l = \begin{bmatrix} \tilde{\Phi}^l \circledast \tilde{\psi}^l_{1,1} & \cdots & \tilde{\Phi}^l \circledast \tilde{\psi}^l_{1,q_l} \\ \vdots & \ddots & \vdots \\ \tilde{\Phi}^l \circledast \tilde{\psi}^l_{q_{l-1},1} & \cdots & \tilde{\Phi}^l \circledast \tilde{\psi}^l_{q_{l-1},q_l} \end{bmatrix}$$

Learned filters

Linear E-D CNN w/ Skipped Connection

$$y = \tilde{B} B^\top x = \sum_i \langle x, b_i \rangle \tilde{b}_i$$

$$\begin{aligned} B &= [E^1 \cdots E^\kappa \quad E^1 \cdots E^{\kappa-1} S^\kappa \quad \cdots \quad E^1 S^2 \quad S^1] \\ \tilde{B} &= [D^1 \cdots D^\kappa \quad D^1 \cdots D^{\kappa-1} \tilde{S}^\kappa \quad \cdots \quad D^1 \tilde{S}^2 \quad \tilde{S}^1] \end{aligned}$$

more redundant expression

$$S^l = \begin{bmatrix} I_{m_{l-1}} \circledast \psi^l_{1,1} & \cdots & I_{m_{l-1}} \circledast \psi^l_{q_l,1} \\ \vdots & \ddots & \vdots \\ I_{m_{l-1}} \circledast \psi^l_{1,q_{l-1}} & \cdots & I_{m_{l-1}} \circledast \psi^l_{q_l,q_{l-1}} \end{bmatrix} \qquad \tilde{S}^l = \begin{bmatrix} I_{m_{l-1}} \circledast \tilde{\psi}^l_{1,1} & \cdots & I_{m_{l-1}} \circledast \tilde{\psi}^l_{1,q_l} \\ \vdots & \ddots & \vdots \\ I_{m_{l-1}} \circledast \tilde{\psi}^l_{q_{l-1},1} & \cdots & I_{m_{l-1}} \circledast \tilde{\psi}^l_{q_{l-1},q_l} \end{bmatrix}$$

Learned filters

Deep Convolutional Framelets

Perfect reconstruction

$$x = \tilde{B}B^\top x = \sum_i \langle x, b_i \rangle \tilde{b}_i$$

Frame conditions

w/o skipped connection

$$\tilde{\Phi}^l \Phi^{l\top} = \alpha I_{m_{l-1}}, \ \Psi^l \tilde{\Psi}^{l\top} = \frac{1}{r\alpha} I_{rq_{l-1}}$$

w skipped connection

$$\tilde{\Phi}^l \Phi^{l\top} = \alpha I_{m_{l-1}}, \ \Psi^l \tilde{\Psi}^{l\top} = \frac{1}{r(\alpha+1)} I_{rq_{l-1}}$$

Ye et al, SIIMS 2018; Ye et al, ICML 2019

Deep Convolutional Framelets

Perfect reconstruction

$$x = \tilde{B}B^\top x = \sum_i \langle x, b_i \rangle \tilde{b}_i$$

Frame conditions

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w skipped connection

$$\tilde{\Phi}^l \Phi^{l\top} = \alpha I_{m_{l-1}}, \quad \Psi^l \tilde{\Psi}^{l\top} = \frac{1}{r(\alpha+1)} I_{rq_{l-1}}$$

Ye et al, SIAM J. Imaging Science, 2018

Role of ReLUs?
Generator for Multiple Expressions

$$y = \tilde{B}(x) B(x)^\top x = \sum_i \langle x, b_i(x) \rangle \tilde{b}_i(x)$$

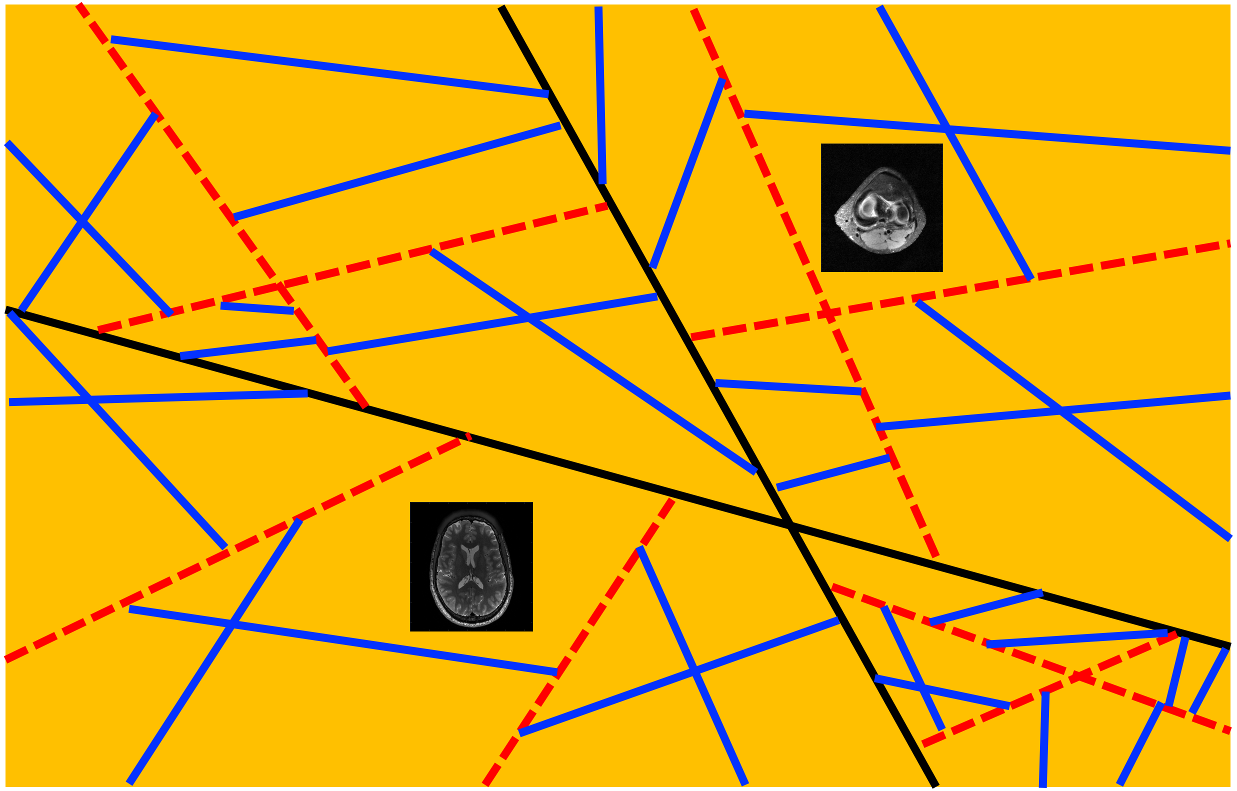
$$\begin{aligned} B(x) &= E^1 \Sigma^1(x) E^2 \cdots \Sigma^{\kappa-1}(x) E^\kappa, \\ \tilde{B}(x) &= D^1 \tilde{\Sigma}^1(x) D^2 \cdots \tilde{\Sigma}^{\kappa-1}(x) D^\kappa \end{aligned}$$

$$\Sigma^l(x) = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{m_l} \end{bmatrix}$$

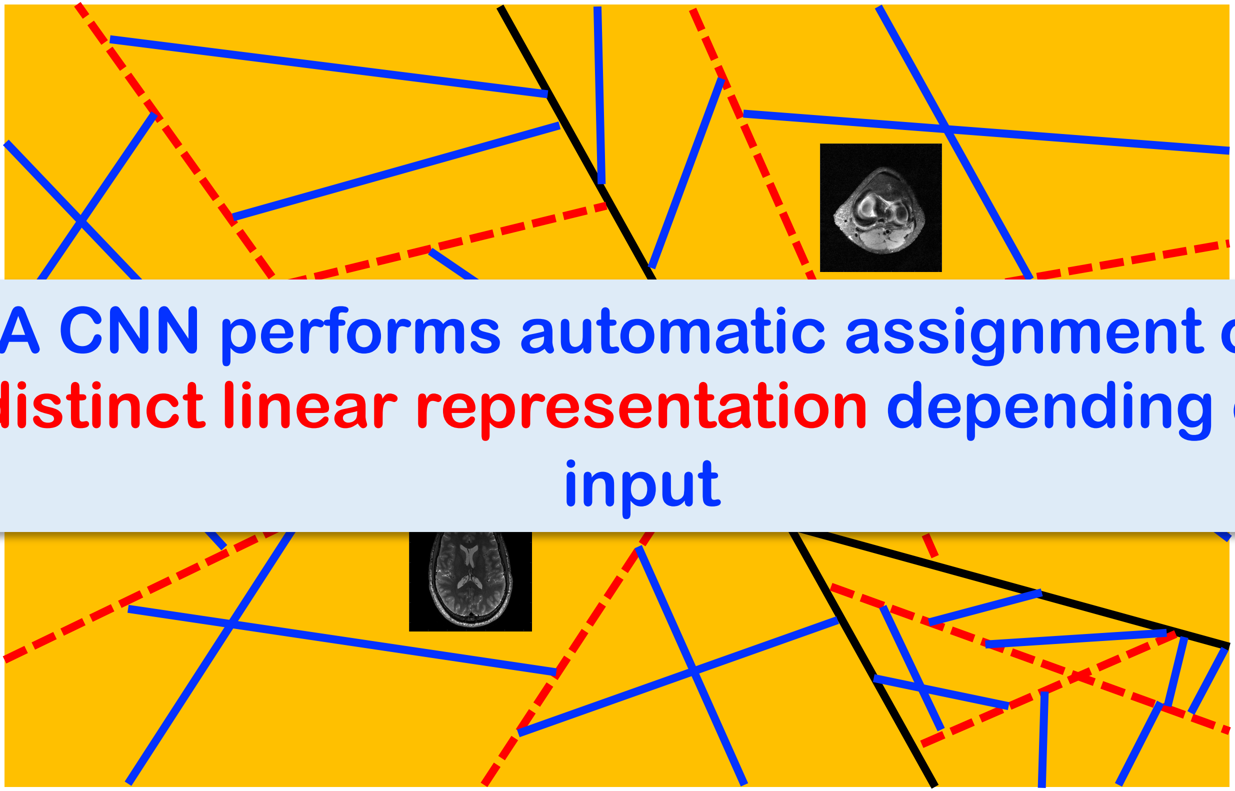
Input dependent {0,1} matrix

--> Input adaptivity

Input Space Partitioning for Multiple Expressions

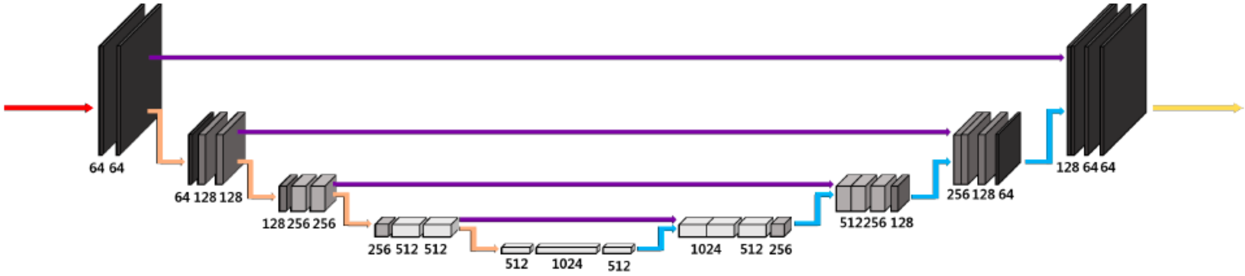


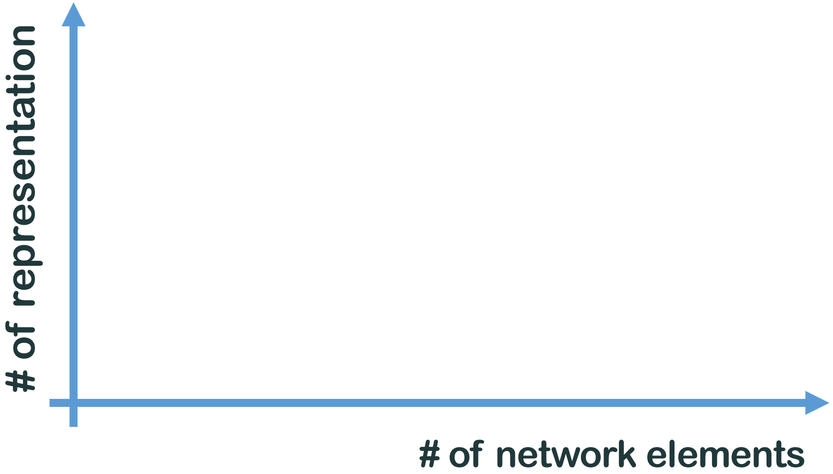
Input Space Partitioning for Multiple Expressions



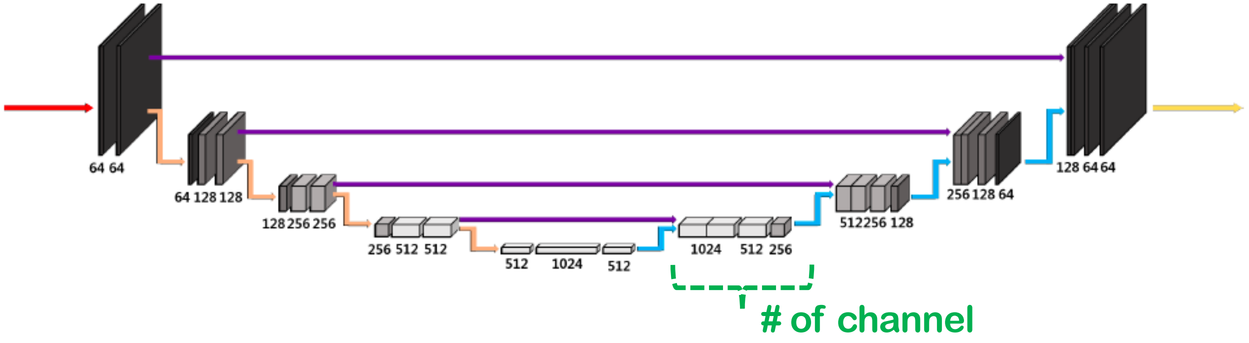
A CNN performs automatic assignment of distinct linear representation depending on input

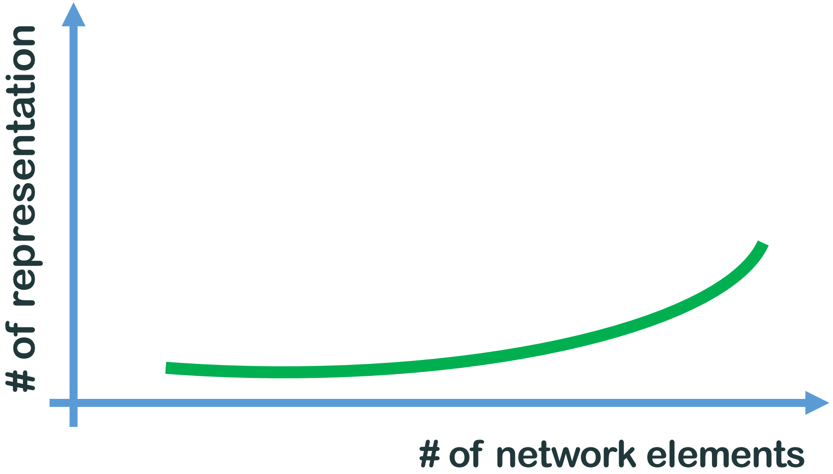
Expressivity of E-D CNN



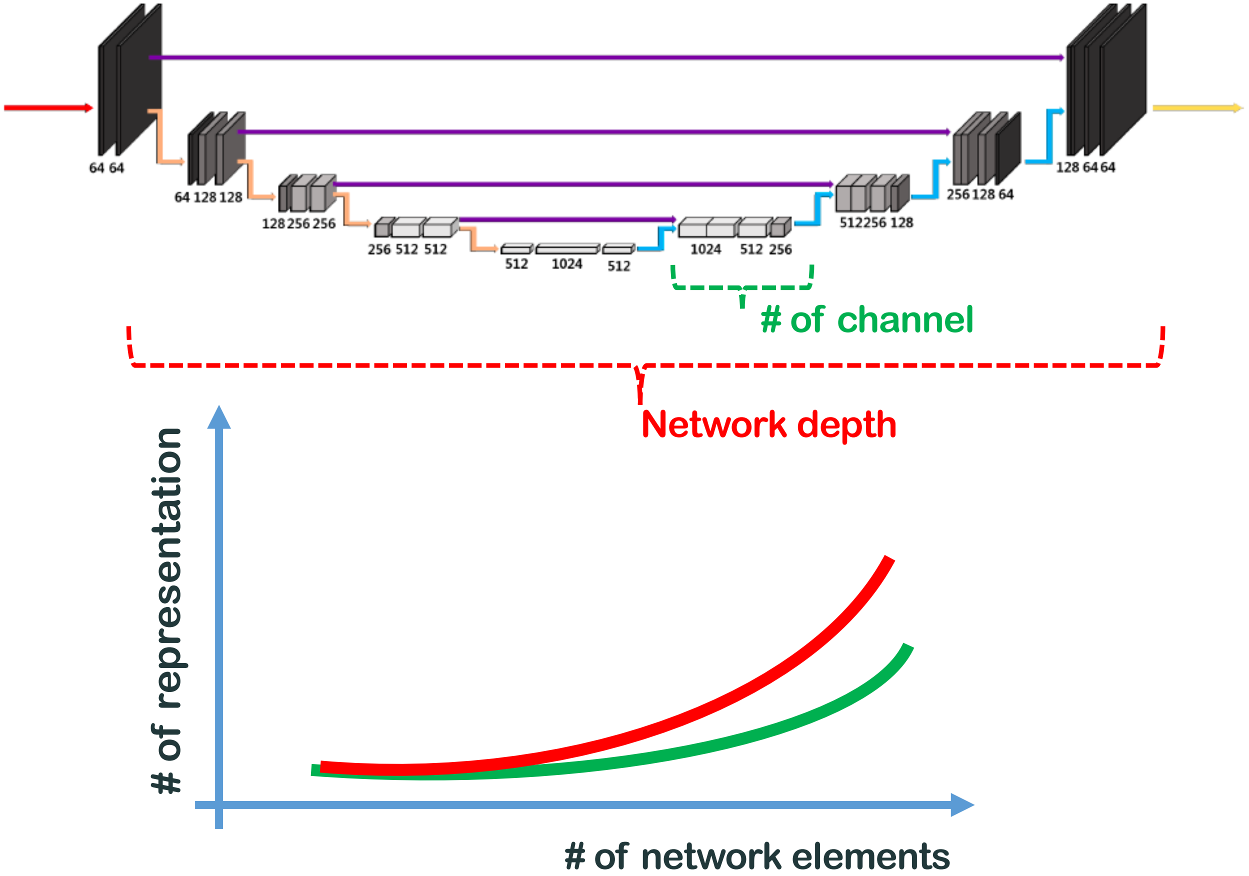


Expressivity of E-D CNN

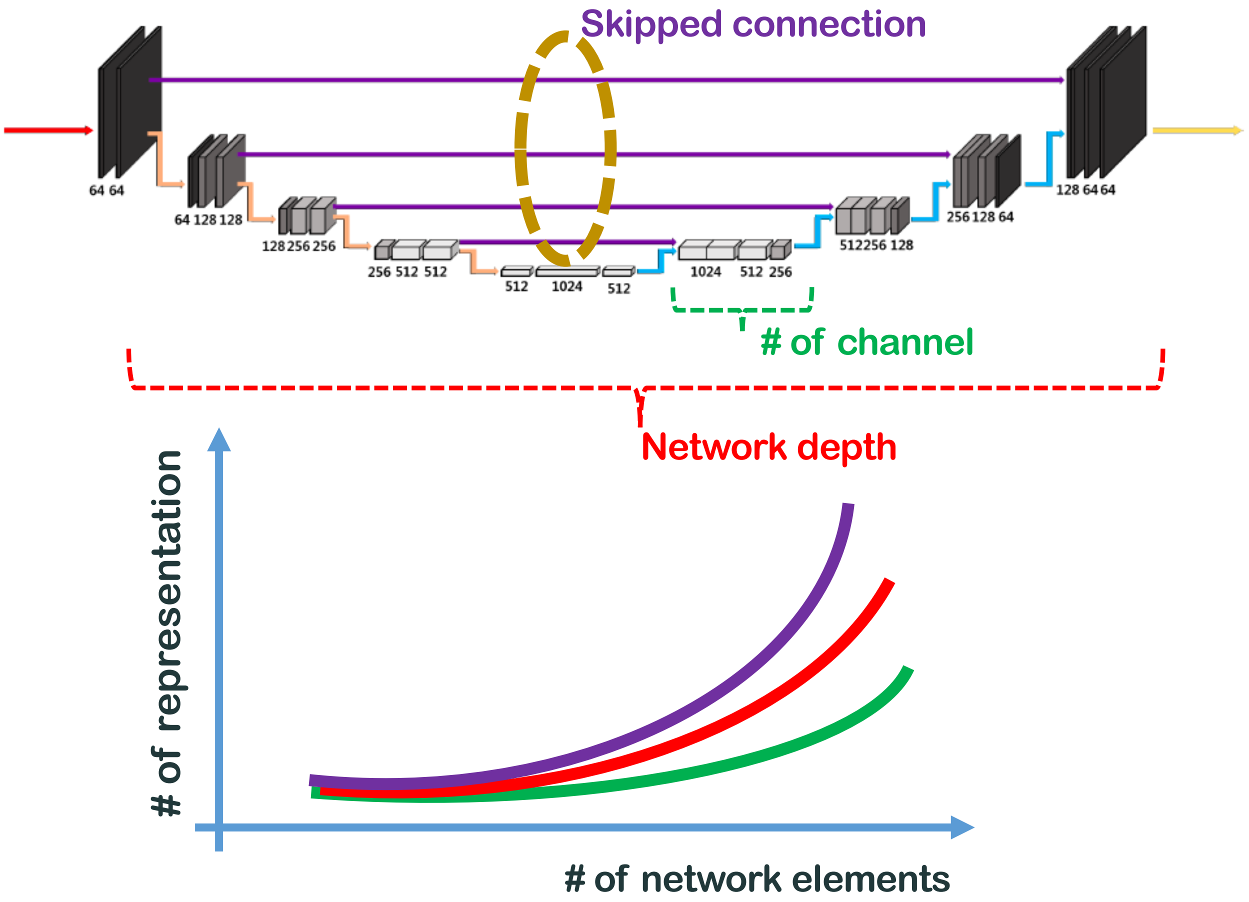




Expressivity of E-D CNN



Expressivity of E-D CNN



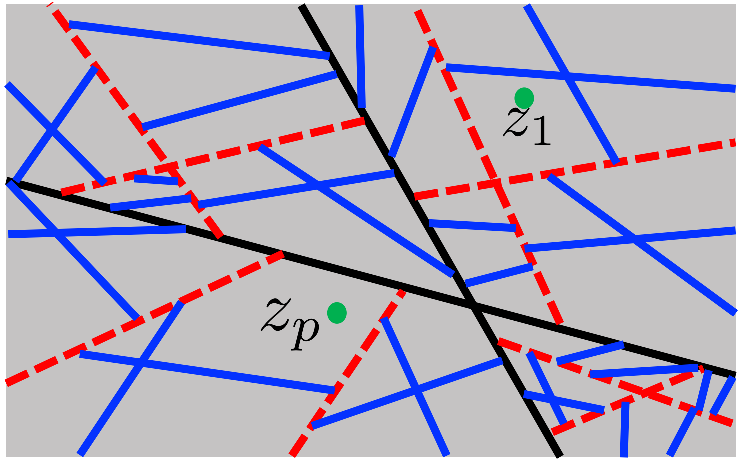
Lipschitz Continuity

Related to the generalizability

$$\|F(\mathbf{W}, x^{(1)}) - F(\mathbf{W}, x^{(2)})\|_2 \leq K\|x^{(1)} - x^{(2)}\|_2$$

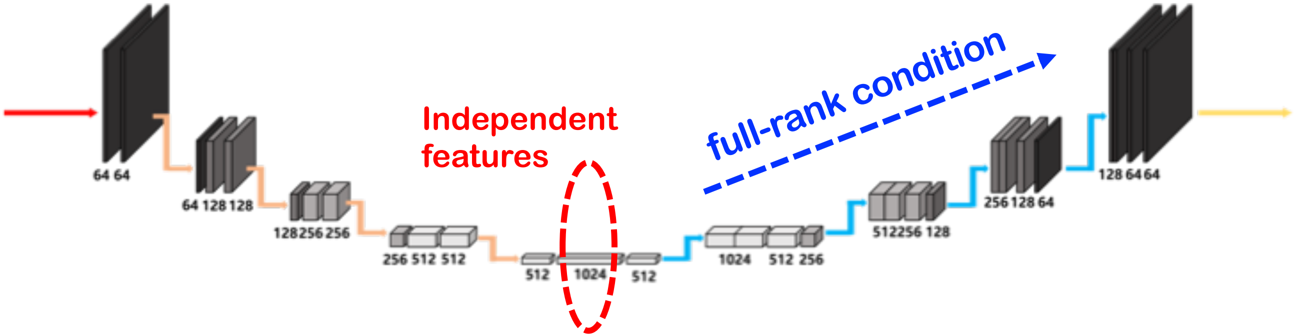
$$K = \max_p K_p, \quad K_p = \|\tilde{B}(z_p)B(z_p)^\top\|_2$$

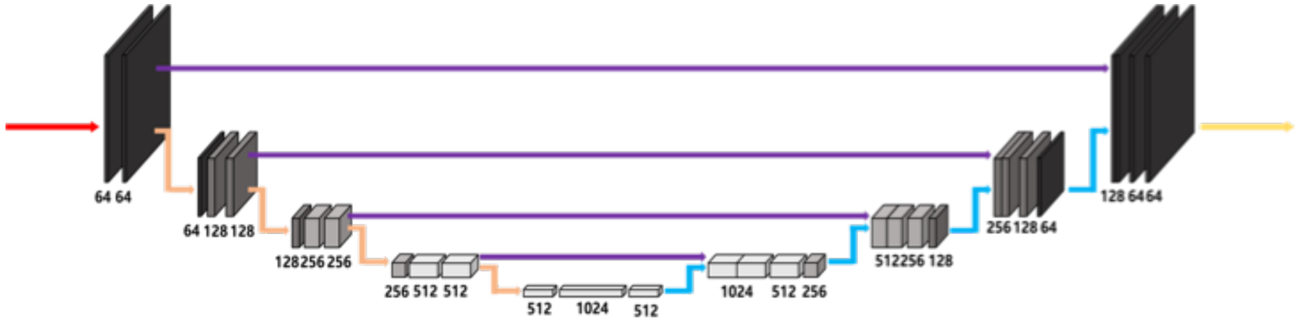
Dependent on the Local Lipschitz



Benign Optimization Landscape

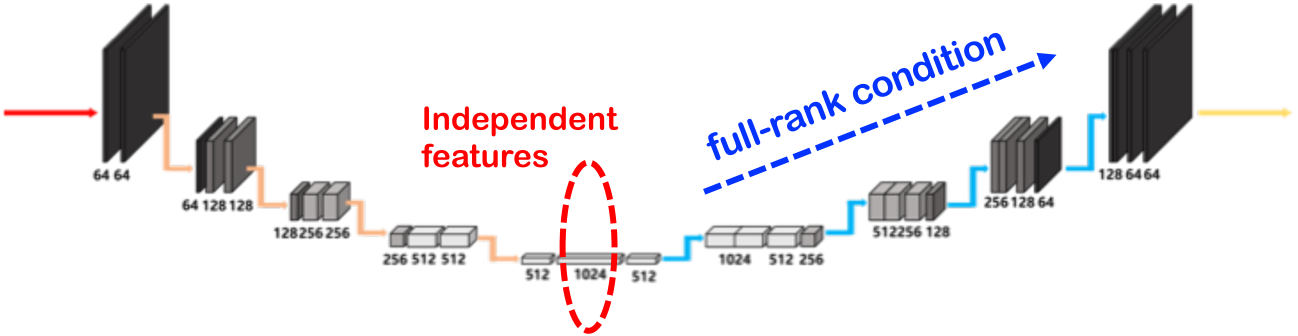
Nguyen, et al, ICML, 2018



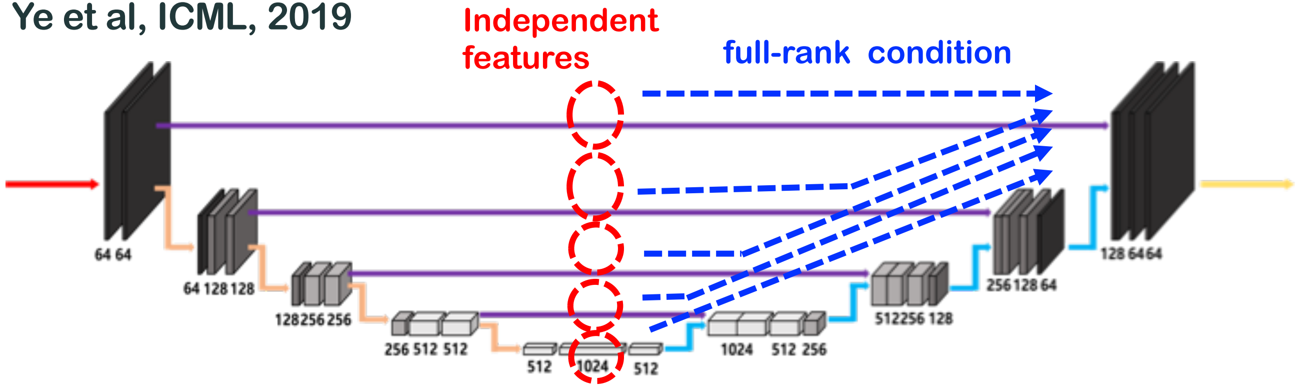


Benign Optimization Landscape

Nguyen, et al, ICML, 2018

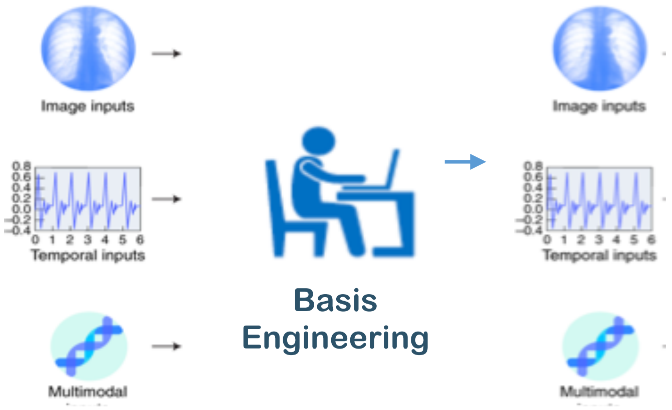


Ye et al, ICML, 2019

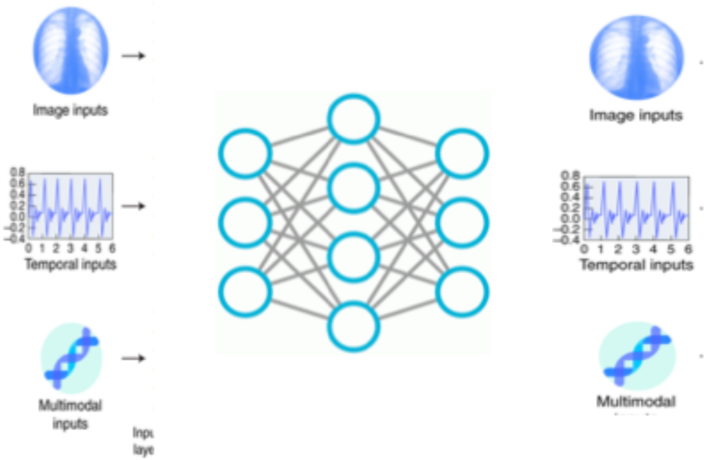


Regularized Recon vs. Deep Recon

Classical Regularized Recon
(basis engineering)



Deep Recon
(no basis engineering)



Summary

- Deep learning is a novel signal representation using combinatorial framelets
- ReLUs generate multiple linear representation by partitioning the input space
- Local Lipschitz controls the global Liptschiz continuity
- Skipped connection improves the optimization landscape
- Black-box nature of neural networks have been being unveiled.

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