Nonnegative Matrix Factorization for Dynamic Computed Tomography

Pascal Fernsel[†], Andreas Hauptmann[‡,*], Simon Arridge[*]

University of Bremen[†], *University of Oulu*[‡], *University College London*[*]

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Dynamic Computed Tomography (DCT)

$$Y(\theta, s, t) := (\mathcal{R}_{I(t)} X(\boldsymbol{x}, t))(\theta, s)$$

- $I(t) \ni (\theta, s)$ measurements with $|I(t)| = n$ for all t
- **Note**: $\mathcal{R}_{I(t)} f \geq 0$ for $f \geq 0$

Nonnegative Matrix Factorization (NMF)

- **Given**: Matrix $\boldsymbol{X} \in \mathbb{R}^{N \times T}_{\geq 0}$
- **Find**: Matrices $\boldsymbol{B} \in \mathbb{R}^{N \times K}_{\geq 0}$, $\boldsymbol{C} \in \mathbb{R}^{K \times T}_{\geq 0}$ with $K \ll \min\{N, T\}$, s.t.

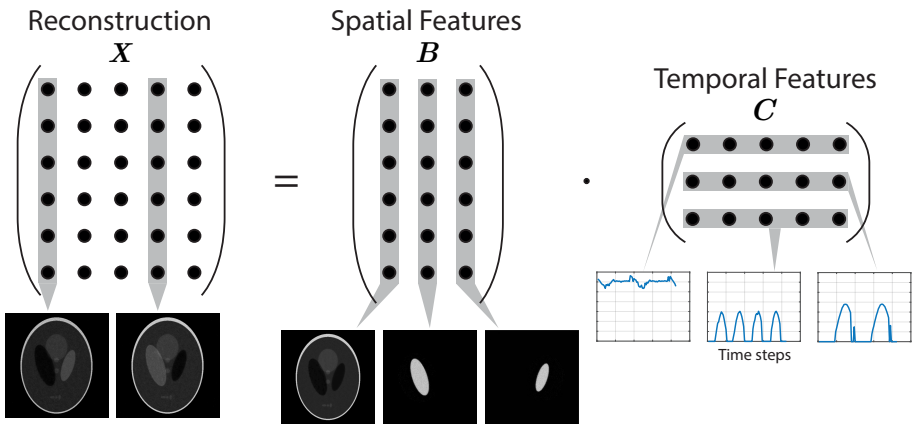
$$\boldsymbol{X} \approx \boldsymbol{B}\boldsymbol{C} = \sum_{k=1}^{K} \boldsymbol{B}_{\bullet,k} \boldsymbol{C}_{k,\bullet}$$

Solving the DCT Problem and Extract Dynamic Features via NMF

NMF Model BC-X

$$\min_{\boldsymbol{B},\boldsymbol{C},\boldsymbol{X} \geq 0} \Bigg\{ \underbrace{\sum_{t=1}^{T} \frac{1}{2} \|\boldsymbol{R}_t \boldsymbol{X}_{\bullet,t} - \boldsymbol{Y}_{\bullet,t}\|_2^2}_{\text{DCT Problem}} + \underbrace{\frac{\alpha}{2} \|\boldsymbol{B}\boldsymbol{C} - \boldsymbol{X}\|_F^2}_{\text{Feature Extraction}} + \underbrace{\sum_{\ell=1}^{L} \gamma_\ell \varphi_\ell(\boldsymbol{B}, \boldsymbol{C}, \boldsymbol{X})}_{\text{Regularization}} \Bigg\}$$

Feature Extraction



Ground Truth

Reconstruction
(Example: BC-X for 8 Angles)

Comparison of Methods
(1% noise, 1500 Iterations)

	BC			BC-X			PCA		
#Ang	PSNR	SSIM	Time [s]	PSNR	SSIM	Time [s]	PSNR	SSIM	Time [s]
2	25.846	0.8035	146.07	27.692	0.8784	98.220	21.511	0.5522	118.34
3	29.584	0.8775	153.74	30.369	0.9133	102.17	23.060	0.6037	121.38
4	33.241	0.9285	180.78	33.581	0.9504	112.00	24.593	0.6738	133.42
5	32.904	0.9195	202.21	34.034	0.9444	123.14	25.508	0.6984	144.03
6	35.544	0.9391	212.97	35.151	0.9525	129.14	25.598	0.6807	149.32
8	35.123	0.9351	265.55	35.625	0.9479	149.55	26.790	0.7167	168.77
10	36.898	0.9452	288.07	35.980	0.9446	163.47	27.396	0.7277	184.98
15	37.762	0.9482	382.16	36.592	0.9391	204.87	28.313	0.7384	224.71

Feature Extraction
(Example: BC-X for 8 Angles)